HORIZONTAL PIPELINE FLOW OF OIL-WATER MIXTURES

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HORIZONTAL PIPELINE FLOW OF OIL-WATER MIXTURES

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BY

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V_W = 0.162 FT PER SEC

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V_W = 0.206 FT PER SEC

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V_W = 0.247 FT PER SEC

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ABSTRACT

The flow characteristics of a white mineral oil, Kremol 70, in the presence of water were examined in a smooth one-inch pipe. The input oil-water volume ratios were varied from 0.1 to 10.0 for 13 superficial water velocities ranging from 0.116 ft per sec to 3.55 ft per sec.

A review of the literature indicated that the flow of two liquids was most analogous to gas-liquid flow.

A general analysis of the flow of two immiscible liquids between wide parallel plates is made and the equations derived for pressure drop are modified and put in a form similar to those for circular pipes. For a parallel plate system of separation 0.1 feet and width 1.0 feet, a parallel plate friction factor is calculated and plotted against the superficial water velocity for eight input oil-water volume ratios from 0.0 to 13.4.

The pressure drop flow rate data in the present experimental study are correlated for the rates and ratios studied in terms of a friction factor based on the water properties and a superficial water velocity. The general shape of this experimental set of curves is in good agreement with the theoretical set.

Flow patterns of the mixtures were examined photographically and qualitatively related to pressure drop measurements. The three types of flow are: bubble, laminar and mixed.

The theoretical development indicates that hold-

up for laminar liquid-liquid flow between wide parallel plates is a function of only the liquid input ratio and the viscosity. The experimental results confirm this prediction in the laminar region but indicate that superficial water velocity is also a variable in the turbulent region.

INTRODUCTION

Studies concerned with the development of the Athabasca oil sands have from time to time revealed interesting problems in materials handling. Not the least of these is that associated with the handling of heavy crude oil in the presence of water.

Clark (4) observed that under some conditions the flow of the heavy Athabasca oil through a pipe was aided by the inclusion or injection of water into the flowing stream. It appeared that the inside of the pipe was wetted by water instead of the viscous oil and a lubrication effect was the result. Clark and Shapiro (3) of Socony Vacuum Oil Company noticed the same effect and patented a process for injection of wetting agents with water into crude oil pipelines. The wetting agents were added to prevent destruction of the water film in the pipe.

Other reports of water injection into crude oil pipelines substantiated the observations but it appears that little detailed study has been given to the oil-water system operative in these instances.

The present project was begun with a view to laying the foundations for a comprehensive study of oil-water flow. The flow of oil-water systems in porous media has been extensively studied (15) but the type of flow encountered is not analogous to pipeline transportation of two fluids. This work does however, give some qualitative ideas on wettability of pipe surfaces, and might provide some ideas for further

extension of pipeline work.

Pipeline flow of the various systems; solid-liquid, liquid-liquid and gas-liquid has received varying amounts of attention from different investigators. Flow of solid-liquid systems has been quite extensively studied and many correlations are available. Most of these correlations attempt to handle the problem from a rheological aspect and assume a uniform flowing material for all velocities. Since the flow pattern in gas-liquid and in liquid-liquid flow is constantly changing the solid-liquid type of approach is not too useful for liquid-liquid flow. Literature on the flow of two immiscible liquids in a pipeline does not exist at the present time. However, a great deal of work has been done on gas-liquid flow and because this type of flow more closely approximates liquid-liquid flow than any other, a general survey of the literature in the field is discussed below.

The general study of two phase gas-liquid flow has been mainly concerned with the effect of certain variables on flow pattern, pressure drop and hold-up. The pertinent variables examined in previous work are:

(1) Density of each phase

(2) Viscosity of each phase

(3) Flow rate of each phase

(4) Geometry of the system

Since for design purposes it is often necessary to have pressure drop data, a good deal of work has been done to obtain a correlation between the variables mentioned and the

experimental pressure drop data.

An attempt to correlate two-phase flow in a manner similar to single phase flow using a type of Fanning equation has been made by some investigators. Uren et al (23) and Howels (11) both working with gas-liquid systems in vertical pipes suggested that the Fanning equation could be used by assuming an average mixture density, viscosity and velocity. The difficulty in this method of correlation was the evaluation of the properties of the flowing mixture and the lack of attention to hold-up and flow pattern considerations. Lockhart and Martinelli (13) using an analysis of flow patterns made by Martinelli et al (14) have correlated pressure drop for horizontal flow by using two parameters ϕ and X. In the analysis, X is the ratio of the pressure drop per unit length of liquid flowing alone to the pressure drop per unit length of gas flowing alone; ϕ is the ratio of the pressure drop per unit length during two phase flow to the pressure drop per unit length for either phase flowing alone. These two parameters were evaluated for each of the following gas-liquid flow mechanisms:

(1) Both phases in turbulent flow

(2) Both phases in laminar flow

(3) Gas in laminar flow, liquid in turbulent flow

(4) Gas in turbulent flow, liquid in laminar flow

Jenkins (12) and Bergelin (2) tried some correlations with Martinelli and Lockhart parameters and found that while their data did fit the ϕ and X correlations in general, there

was a spread ± 30 per cent. Baker (1) obtained data for flow through pipes four to 10 inches in diameter and found the Martinelli type correlation to be inadequate. White and Huntington (24) presented an empirical relation which predicted the pressure drop in a gas-liquid system flowing in a horizontal direction from a knowledge of the flow rates, physical properties and pipe diameter. This correlation was limited to the flow patterns that are stable in nature; this excluded slug and stratified flow types. The correlation was not checked at high pressures and it did not apply to liquids with viscosity less than 120 centipoise.

Chenowith and Martin (8) attempted to correlate Martinelli's and Lockhart's work at higher pressures in the turbulent flow region and found that the predicted pressure drop was 1.4 to 2.5 times the observed pressure drop.

Work done at the University of Alberta, by Govier and his associates, on vertical gas-liquid flow has led to a separation of the hydrostatic head component from the irreversibility component. This was accomplished by Radford (17) who applied a mechanical energy balance to the two flowing fluids and developed a general equation which permitted the separation of the pressure drop into its two components. Radford's (17) correlation made use of a modified Reynolds number based on the gas phase alone and a friction factor calculated by considering only the gas properties and the irreversibility component. Dunn(9) continued this work and proposed a correlation based on the

flowing gas phase using a superficial friction factor and Reynolds number. A superficial friction factor based on the tube liquid properties was successfully correlated with superficial air and water velocities and tube diameter by Short (18). Short's correlation enables a calculation of total pressure drop for vertical flow of air-water mixtures at an average pressure of 36 psi and a range of tube diameters from 0.630 to 2.50 inches.

The above work on vertical flow has been continued by Sullivan (22) in an investigation of the vertical flow of two liquids, water and Varsol.

Some work by Sobocinski and Huntington (19) on three phase flow comes closest to the present study in type of system investigated but doesn't suggest any correlation for liquid-liquid flow.

Examination of gas-liquid vertical flow seems to indicate that the following types of flow patterns exist.

(1) A dispersed phase of gas bubbles throughout the liquid

(2) A dispersed phase of liquid droplets throughout the gas

(3) Alternate slugs of gas and liquid

(4) A core of gas flowing through an annular ring of liquid

White and Huntington (24) report the series of flow patterns described below for gas-liquid horizontal flow.

(1) Stratified flow

(2) Ripple flow

(3) Slug flow

(4) Wave flow

Although Moore and Wilde (16) were among the first investigators to recognize the presence of slippage leading to hold-up in two phase flow, Govier, Radford and Dunn (10) provided the best method for handling and defining hold-up. It is defined as the ratio between the input mixture and the in situ or test section mixture and may be related to the average velocities of the phases as follows:

$$H_R = \frac{V_G - V_L}{V_L} + 1$$

where

H_R = Hold-up ratio, dimensionless

V_G = Average gas phase velocity, ft per sec

V_L = Average liquid phase velocity, ft per sec

It appears that to understand and to interpret experimental work on two liquids flowing in a pipe that information on pressure drop, flow pattern and hold-up must be obtained through the use of a pilot pipeline.

THEORY

A theoretical analysis of the laminar flow of two immiscible liquids would make it possible to predict pressure drops and hold-up ratios for any velocity and any input ratio if the fluid properties were known. To complete a mathematical analysis the relative position of the two streams must be defined and a fixed flow pattern established. The simplest system for this type of analysis pertinent to liquid-liquid flow is the passage of two immiscible liquids through parallel plates of infinite width and finite separation.

A complete analysis of liquid-liquid flow in parallel plate systems has not been described in the literature although Coulson and Richardson (6) and Corcoran et al (5) touched on one small phase of it in a student exercise. The following discussion summarizes a complete analysis of the flow in this system. The detailed development is shown in Appendix D.

Figure 1 is a sketch of the flow system in which parallel plates are orientated in a horizontal position separated by a distance 2a. The liquid-liquid interface is parallel to the plates at a distance y above the lower plate.

In the analysis the following assumptions are made:

(1) The upper layer is less dense than the lower layer.

(2) There is no mixing of the liquids at the interface and the flow is in two distinct layers.

(3) Both liquids are Newtonian and incompressible.

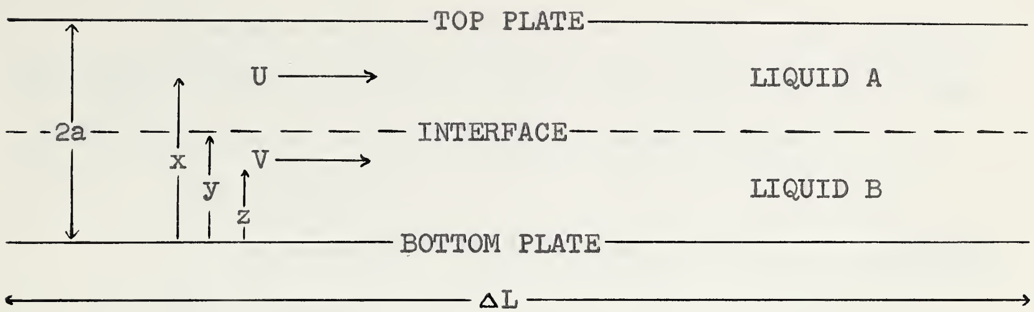


FIGURE 1

NOMENCLATURE

$2a$ = distance between plates, ft

b = width, ft

ΔL = length, ft

y = distance from the bottom plate to the interface

x = distance from the bottom plate to any point in liquid A

z = distance from the bottom plate to any point in liquid B

U = velocity of liquid A, ft per sec

V = veloctiy of liquid B, ft per sec

Q_A = volumetric flow rate of liquid A, ft^3 per sec

Q_B = volumetric flow rate of liquid B, ft^3 per sec

μ_A = viscosity of liquid A, lbs_M per ft-sec

μ_B = viscosity of liquid B, lbs_M per ft-sec

ΔP = total pressure drop over the length ΔL, lbs_F per ft^2

R_x = unit shear force in the liquid at distance x, lbs_F per ft^2

R_z = unit shear force in the liquid at distance z, lbs_F per ft^2

R_i = unit shear force at the interface of the two liquids, lbs_F per ft^2

g_c = dimensional conversion factor, lb_M ft per lb_F sec^2

A separate force balance on liquid A and liquid B and application of Newton's second law yield the following equations since the acceleration is equal to zero.

(i) Liquid A

$$\Delta P\,(x-y)b - R_x \Delta L b + R_i \Delta L b = 0 \;............(1)$$

(ii) Liquid B

$$\Delta P\,(y-z)b - R_z \Delta L b - R_i \Delta L b = 0 \;............(2)$$

Using the relationship between shear force and velocity gradient, $R = \frac{\mu dU}{g_c dx}$, integration and rearrangement of the variables yield:

(i) Liquid A

$$U = \frac{-\Delta P g_c}{\Delta L \mu_A}\left(\frac{x^2}{2} - yx\right) - \frac{R_i g_c}{\mu_A}(x) + C$$

(ii) Liquid B

$$V = \frac{\Delta P g_c}{\Delta L \mu_B}\left(zy - \frac{z^2}{2}\right) - \frac{R_i g_c}{\mu_B}(z) + C$$

The constants are evaluated by substituting in the boundary conditions: when $z = 0$, $V = 0$; when $x = 2a$, $U = 0$.

(i) Liquid A

$$U = \frac{\Delta P g_c}{\Delta L \mu_A}\left(2a^2 - 2ay - \frac{x^2}{2} + yx\right) + \frac{R_i g_c}{\mu_A}(2a-x) \;...(3)$$

(ii) Liquid B

$$V = \frac{\Delta P g_c}{\Delta L \mu_B}\left(zy - \frac{z^2}{2}\right) - \frac{R_i g_c}{\mu_B}(z) \;...............(4)$$

R_i the interface shear force, may be evaluated by considering the system behaviour at the interface. At this point U is equal to V and an expression for R_i can be obtained.

Integration of the velocities over the area of flow enables Q, the flow rate, to be calculated for each liquid. The equation takes the general form:

$$Q_B = \frac{\Delta P b a^3 g_c}{\Delta L \mu_B} \frac{(N_1 \mu_A + N_2 \mu_B)}{(N_3 \mu_A + N_4 \mu_B)} \quad \text{..................(5)}$$

The constants N_1, N_2, N_3 and N_4 are functions of the in situ oil-water volume ratio.

Confirmation of the expression for Q--the flow as a function of pressure drop, viscosity of the liquids and the in situ ratio of the flowing liquids--may be obtained by comparing Q with the expressions developed by Coulson and Richardson (6) and by Streeter (20) for related flow systems. Inserting the limits to evaluate the constants for a one-to-one in situ ratio gives:

$$Q_A = \frac{\Delta P b a^3 g_c}{\Delta L \mu_A 12} \frac{(7\mu_A + \mu_B)}{(\mu_A + \mu_B)} \quad \text{......................(6)}$$

which is in exact agreement with the Coulson and Richardson expression.

Streeter developed the expression:

$$Q = \frac{2 \Delta P b a^3 g_c}{3 \mu \Delta L} \quad \text{...............................(7)}$$

for a single liquid flowing between two parallel plates. Applied to the present two liquid system Q was evaluated for

each liquid for in situ ratios ranging from 0.1 to 10.0. The viscosity of A was then set equal to the viscosity of B and the two flow rates added. In all cases the calculated Q was equal to that derived by Streeter.

Equation 5 expresses the flow rate in terms of the geometry of the system, the pressure drop and viscosity, for a parallel plate flow system. Because the majority of fluid flow takes place in circular conduits it is desirable to adapt this equation for use with a pipe.

Since the velocity distribution for two liquids flowing in a pipe is complex it is extremely difficult to handle this problem in the same manner, but it is felt that some similarity should exist in the general relationship under laminar flow conditions. To facilitate a comparison the derived equations are expressed in terms of a "friction factor" and superficial velocity. For the case of a single liquid in a given pipe the superficial velocity is directly proportional to Reynolds number.

For convenience the friction factor will be based on the properties of a single liquid and is described below:

The conventional friction factor is defined as follows:

$$f = \frac{\Delta P \, D g_c}{2 \Delta L V^2 \rho} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

For parallel plates of width b and separation 2a, $V = \frac{Q}{2ab}$ where 2ab is the area of flow. 2a may be arbitrarily used instead of D in Equation (8) yielding

$$f = \frac{\Delta P a g_c}{\Delta L V^2 \rho}$$

Accordingly f_{pp} may be defined as: $\dfrac{\Delta P 4 a^3 b^2 g_c}{\Delta L\ Q^2 \rho}$ ....(9)

The general equation for Q takes the form:

$$Q_B = \frac{\Delta P b a^3 g_c}{\Delta L \mu_B} \frac{(N_1 \mu_A + N_2 \mu_B)}{(N_3 \mu_A + N_4 \mu_B)}$$

For any two given liquids this becomes $\Delta P = \dfrac{Q \Delta L}{b a^3 K g_c}$

where $K = \dfrac{1}{\mu_B} \dfrac{(N_1 \mu_A + N_2 \mu_B)}{(N_3 \mu_A + N_4 \mu_B)}$ ....(10)

Where N_1, N_2, N_3 and N_4 are functions of the in situ oil-water volume ratio.

Combining this with Equation (9)

$$f_{pp} = \frac{4b}{\rho K Q} \quad \text{.................................(11)}$$

The values of the flow rate, Q, which will assure that flow is in the laminar region can be calculated by using a criterion for laminar flow between parallel plates described by Walker (25).

Walker defined Reynolds number for parallel plates as follows

$$Re = \frac{4aV\rho}{\mu} \quad \text{..................................(12)}$$

For water this becomes

$$Re = 9.26(10)^4\ V4a \quad \text{..........................(13)}$$

Since Walker found experimentally that Reynolds number must be below 2000 for laminar flow, the upper limit on V4a for water flow must be 0.0216.

To enable the construction of a theoretical correlation plot of friction factor versus a superficial velocity the geometry of the system must be fixed. Accordingly it was decided to consider a parallel plate system of width one foot and separation 0.1 feet. The equation for the parallel plate friction factor now becomes

$$f_{pp} = \frac{4}{\rho K Q} \qquad (14)$$

By considering flow rates of $1(10)^{-4}$, $5(10)^{-4}$ and $10(10)^{-4}$ ft^3 per sec which are well in the laminar region as checked by Equation (13) and by evaluating K for seven in situ ratios from 0.0 to 10.0, f_{pp} was calculated using water properties and tabulated for the three water rates in Table D 1V.

From these calculated data Figure 2 was prepared showing a plot of the parallel plate friction factor, f_{pp}, versus V_w, the superficial water velocity. The curves in Figure 2 are lines of constant input oil-water volume ratios calculated from the in situ ratios.

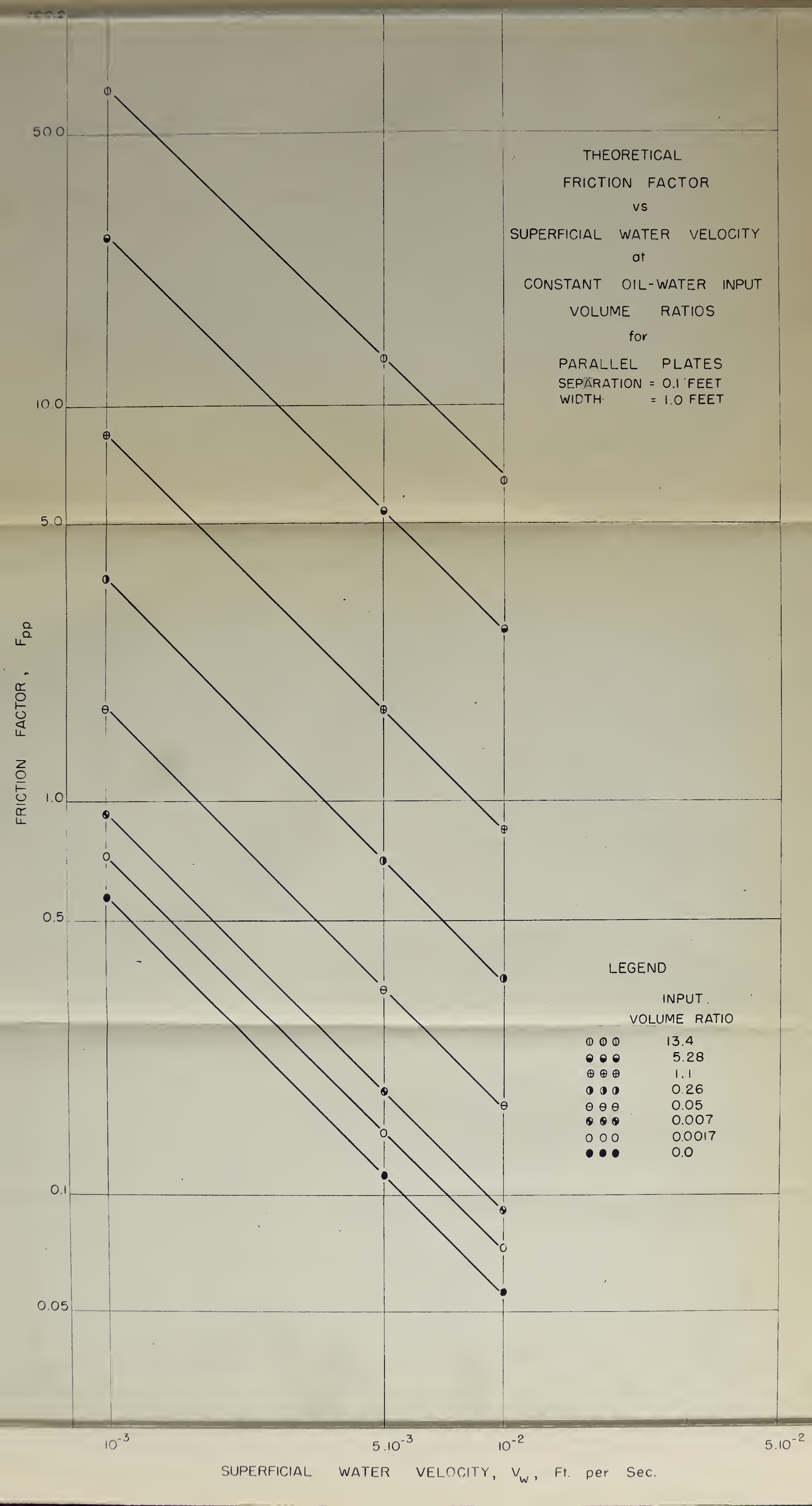
Another important aspect of this theoretical development is the information which it gives regarding hold-up or the ratio of the input flowing mixture to the in situ flowing mixture. To obtain Q_A and Q_B a series of in situ ratios of oil to water were selected and the flow rates were calculated. The input ratio was then obtained by dividing Q_B into Q_A.

A sample calculation for this system of oil and water is shown in Appendix D. From Table D 111, Figure 3

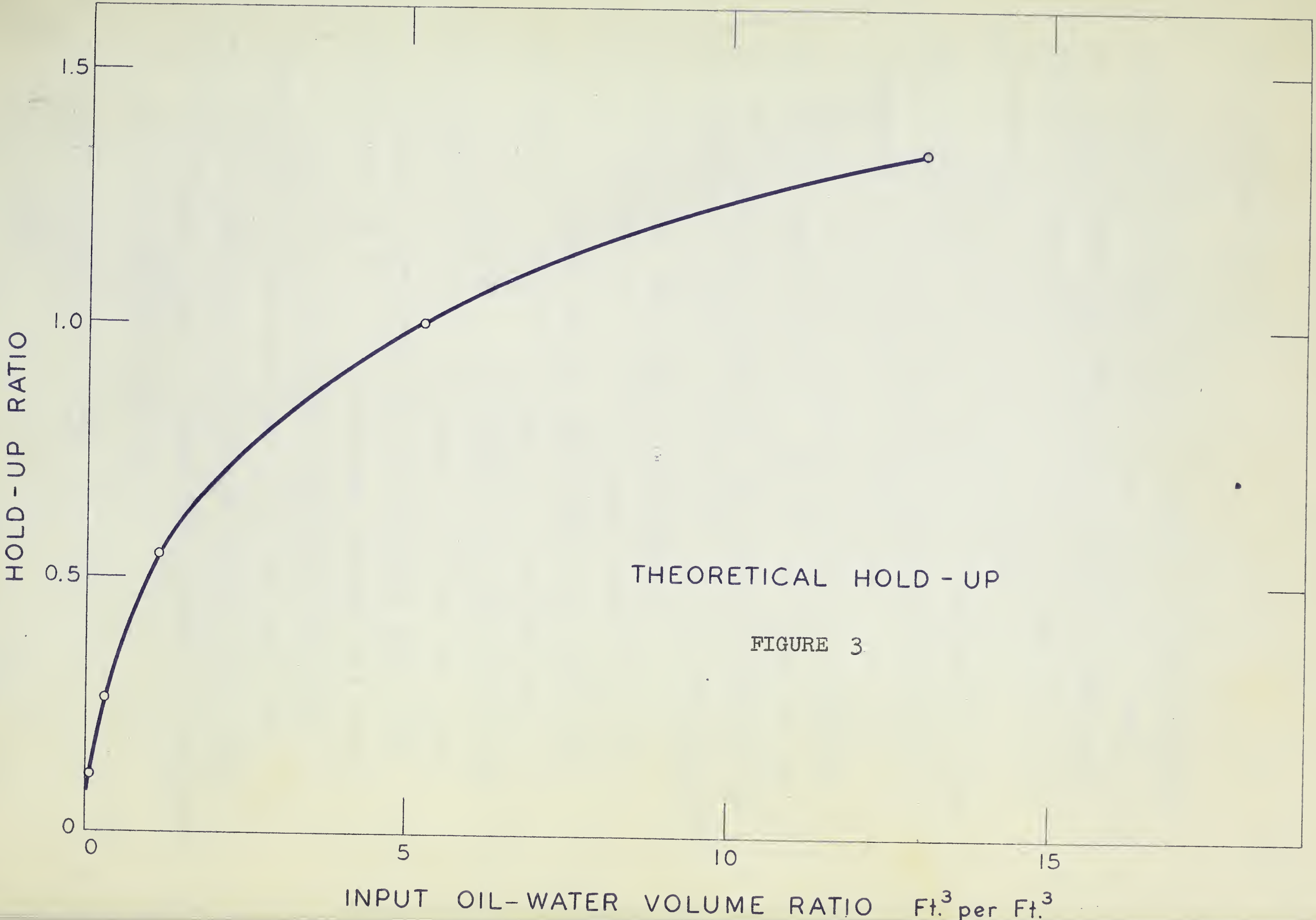
was constructed showing H_R, the hold-up ratio, which is the input oil-water volume ratio divided by the in situ oil-water volume ratio, plotted against the input oil-water volume ratio. This curve shows that for the laminar, two layer flow of Kremol 70 mineral oil and water between two parallel plates the hold-up is a function of the viscosity of the two liquids and their input ratios.

It also indicates that for an input ratio of zero the hold-up ratio approaches zero.

Consideration of the laminar two layer flow of two immiscible liquids between parallel plates has indicated the general form of the friction factor Reynolds number relationship for laminar flow and the effect on it of the input ratio. While the flow of two immiscible phases in a circular pipe presents a much different problem, it is reasonable to anticipate a similar general pattern in the laminar region.



100.0
50.0
10.0
5.0
1.0
0.5
0.1
0.05
FRICTION FACTOR, F_{pp}
THEORETICAL
FRICTION FACTOR
vs
SUPERFICIAL WATER VELOCITY
at
CONSTANT OIL-WATER INPUT
VOLUME RATIOS
for
PARALLEL PLATES
SEPARATION = 0.1 FEET
WIDTH = 1.0 FEET
LEGEND
INPUT
VOLUME RATIO
13.4
5.28
1.1
0.26
0.05
0.007
0.0017
0.0
10^{-3}
5.10^{-3}
10^{-2}
5.10^{-2}
SUPERFICIAL WATER VELOCITY, V_w, Ft. per Sec.



THEORETICAL HOLD-UP

FIGURE 3

EXPERIMENTAL APPARATUS

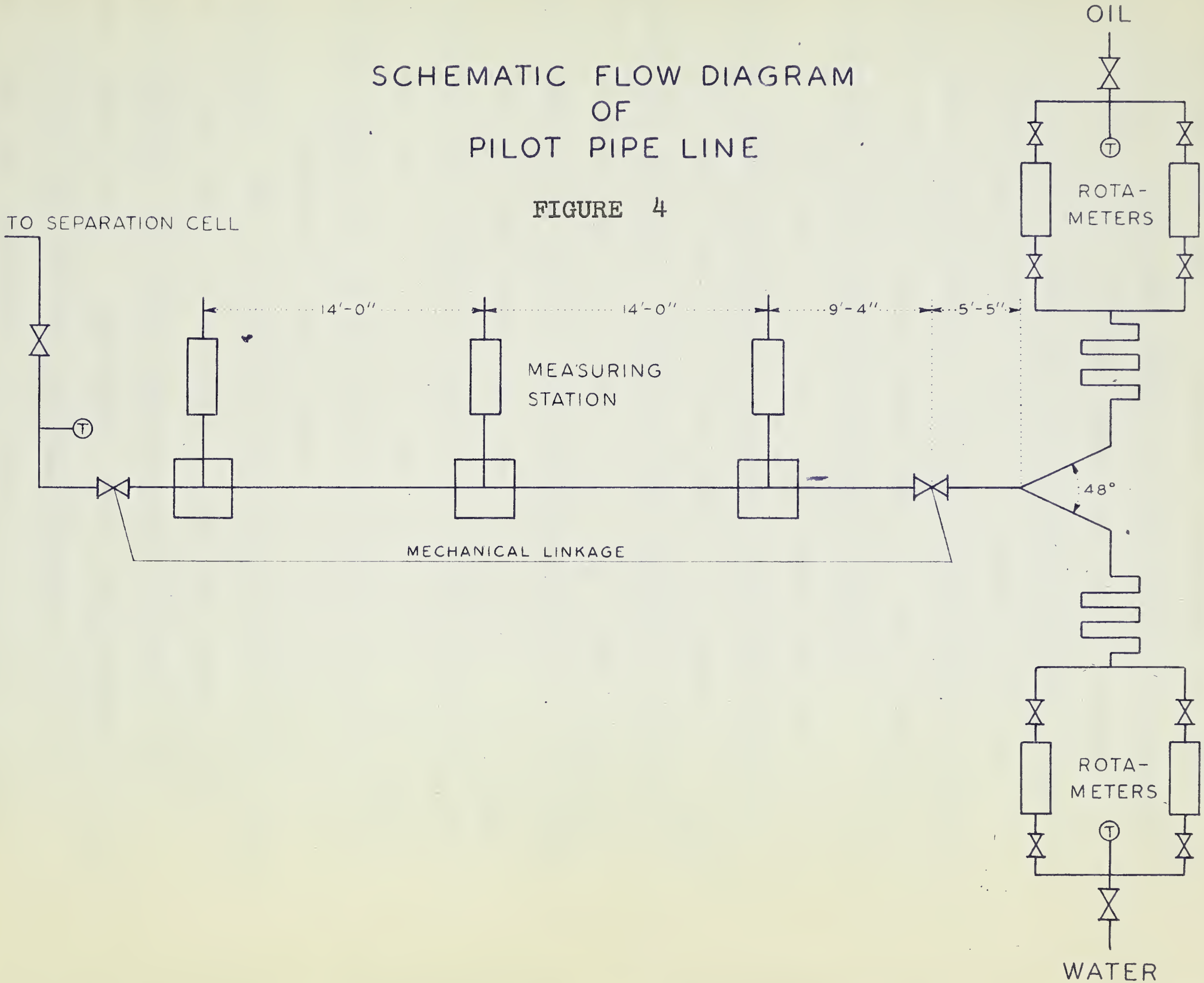
The pilot pipeline was designed to obtain pressure drop and hold-up measurements for two immiscible liquids. The two liquids chosen were water and Kremol 70, a clear light mineral oil with a Saybolt color of 30 +, a specific gravity of 0.834 and a viscosity of 22 cp at 20° C.

A schematic flow diagram of the equipment is shown in Figure 4. The main test section was about 35 feet of transparent cellulose acetate butyrate pipe of one inch diameter supported in a horizontal position by a framework made of light steel sections. The pipe was tapped six feet from the inlet end and about two feet from the discharge end providing a test section of 28.18 feet. A third tapping point was provided half way along the test section.

At the inlet end, the acetate butyrate pipe was joined to the input manifold by an eight foot copper pipe terminating in a welded 48 degree Y. To prevent transmission of vibration from the pumps to the test section and to provide for flexible positioning of the Y, the copper was connected to the input manifold by rubber hose. The discharge end of the test section was attached to a separator by rubber hose. Two roundport quick-closing valves connected by a mechanical linkage were installed at both ends of the plastic pipe for measuring hold-up. The input manifold, pressure measuring system and separator are described in detail in the following sections.

SCHEMATIC FLOW DIAGRAM OF PILOT PIPE LINE

FIGURE 4



Input Manifold

Figure 5 shows the input manifold control panel. It was equipped with two bimetallic thermometers, four rotameters, three pump control switches and three pressure gages. Two of the pressure gages indicated discharge pressure and the other gave test section pressure. The small switch was wired so that the operator could stop both pumps simultaneously. This was installed as a safety feature to prevent pipe rupture when the quick-closing valves were closed for measurements of hold-up.

The metering of the oil and water was done by Fischer Porter rotameters. To give a very wide range of flow rates two rotameters were mounted in parallel for each liquid and the small rotameters were equipped to operate with either a steel or plastic float. The small rotameters had a range from 0.152 to 4.52 Imp gal per min for the water and 0.0495 to 3.60 Imp gal per min for the oil. The large rotameters had a range of 3.54 to 17.3 Imp gal per min for water and 1.07 to 11.7 Imp gal per min for oil. The calibration curves for the rotameters are shown in Appendix C.

Behind the control panel were the pumps and reservoirs. The oil and water were pumped bytwo Moyno pumps driven with V belts by two $7\frac{1}{2}$ hp 3/220/130 standard protected motors. The pumps, manufactured by Robbins and Meyer, were the 2L4 - CDQ models with a displacement of 2.02 U.S. gal per 100 rpm and a working pressure of 300 psi. Both pumps were equipped with safety relief valves because

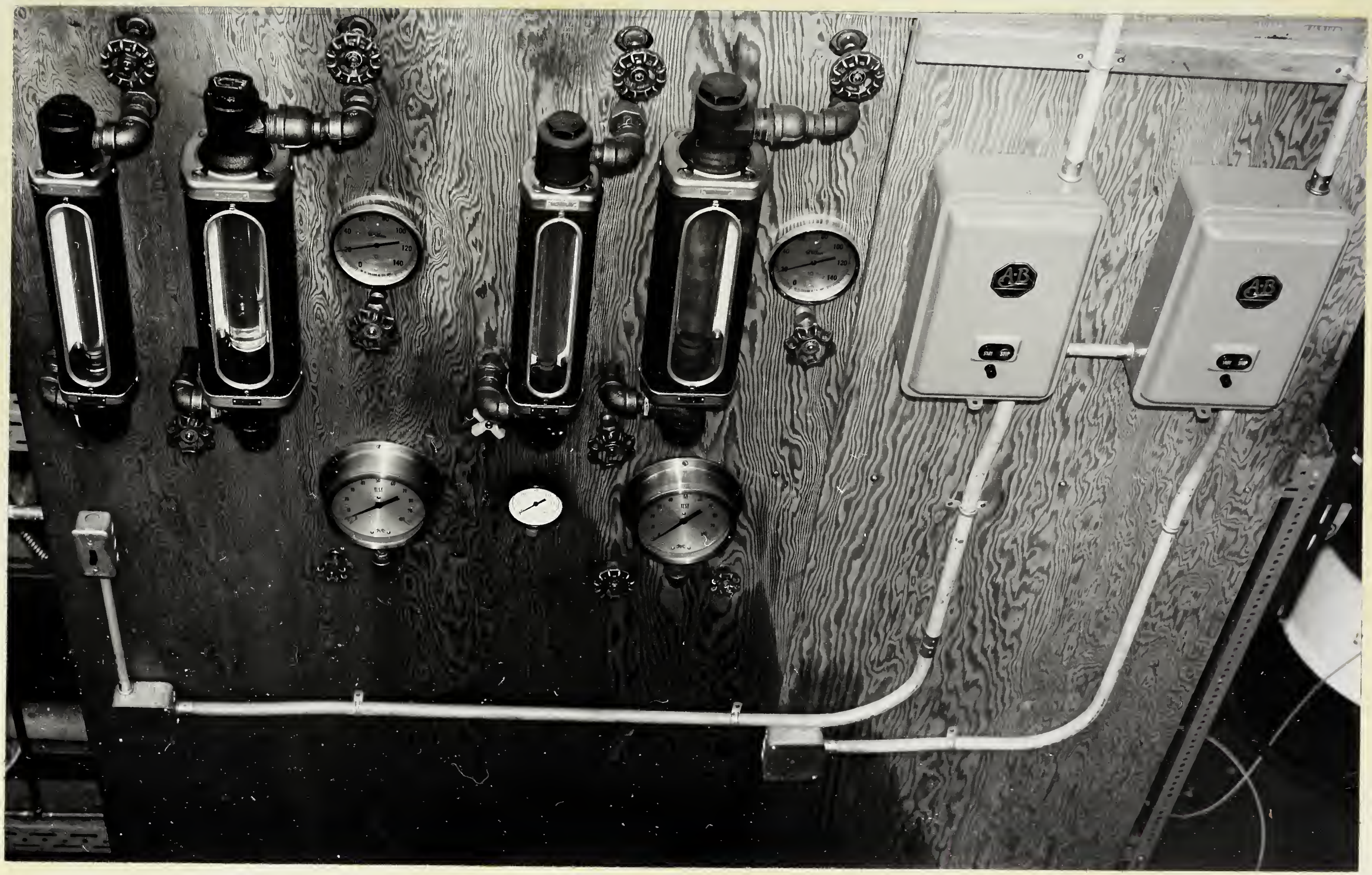


FIGURE 5

PIPELINE CONTROL PANEL

of their positive displacement characteristic. Considerable care was taken with the suction and discharge piping to prevent inclusion of air.

A cooling coil was found to be necessary in the oil reservoir because of heat generated when the oil was recycled. The temperature in the water reservoir was controlled manually by the addition of hot or cold water.

Pressure Measuring System

The pressure measuring system consisted of pressure stations on the flow line, transmission lines, and manometers. The flow equipment in the present investigation was designed to permit the study of oil in the presence of water and because the test section was handling both liquids, special pressure measuring stations were designed. To assure that the manometer transmission lines were filled with a material of uniform density, air was employed as the transmitting fluid. The measuring station as shown in Figure 6 consisted of a block of lucite through which the flow line passed. A hole 1/8 inch in diameter was drilled into the top of the block and through the pipe wall. The hole in the block was enlarged and tapped to take a $\frac{1}{4}$ inch pipe nipple which supported the splash pot with its gage, valve and tubing connections. A measuring scale was mounted on each splash pot. The elevations of the scales were adjusted to the same absolute elevation.

Figure 7 shows the polyethylene tubing connections from the measuring stations to the manometers. The dotted

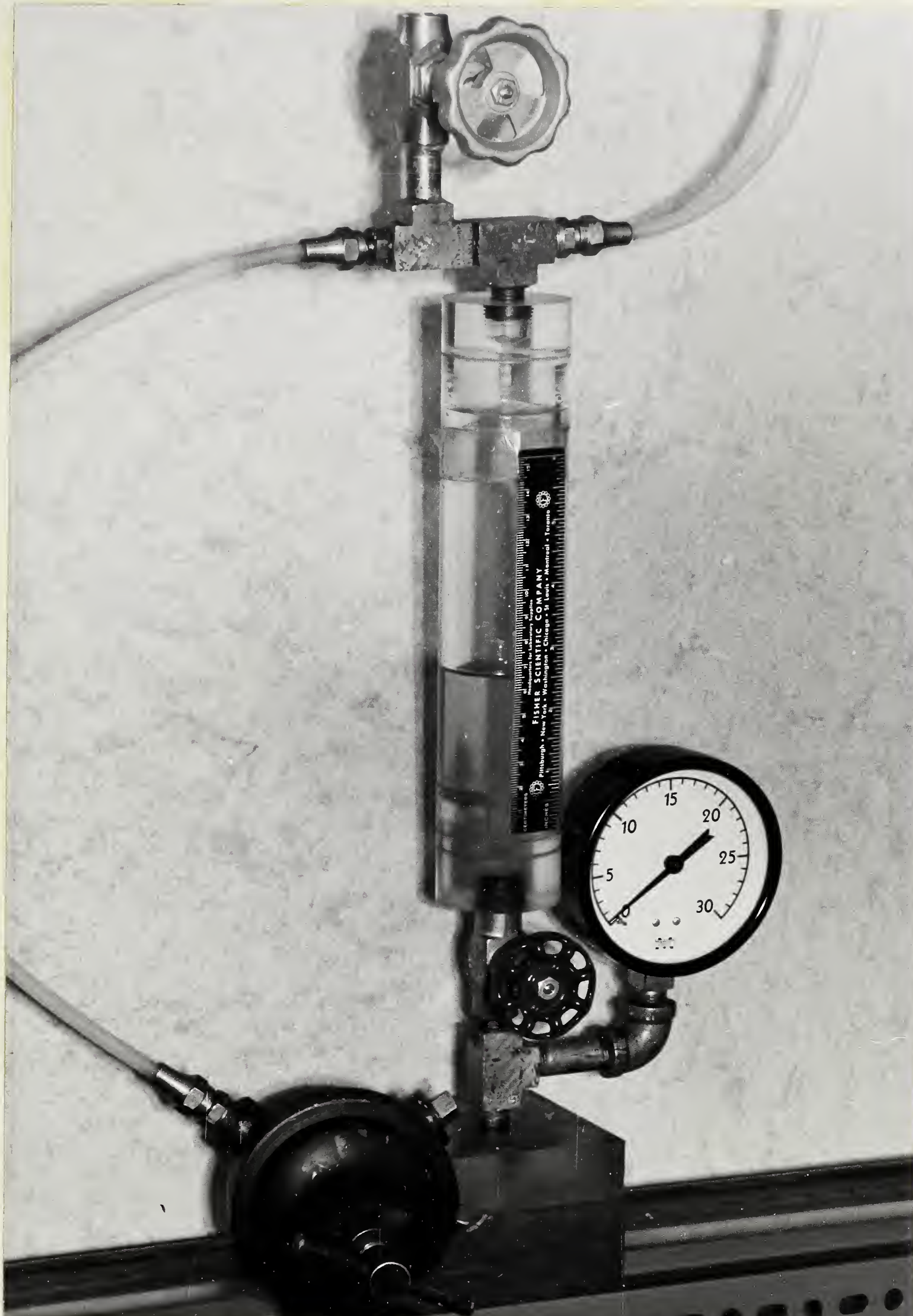


FIGURE 6

PRESSURE MEASURING STATION

lines indicate temporary connections which were installed only as needed. Pressure differentials were measured with three Meriam manometers: one 12 foot mercury manometer, one 12 foot water manometer and one 30 inch inclined water manometer.

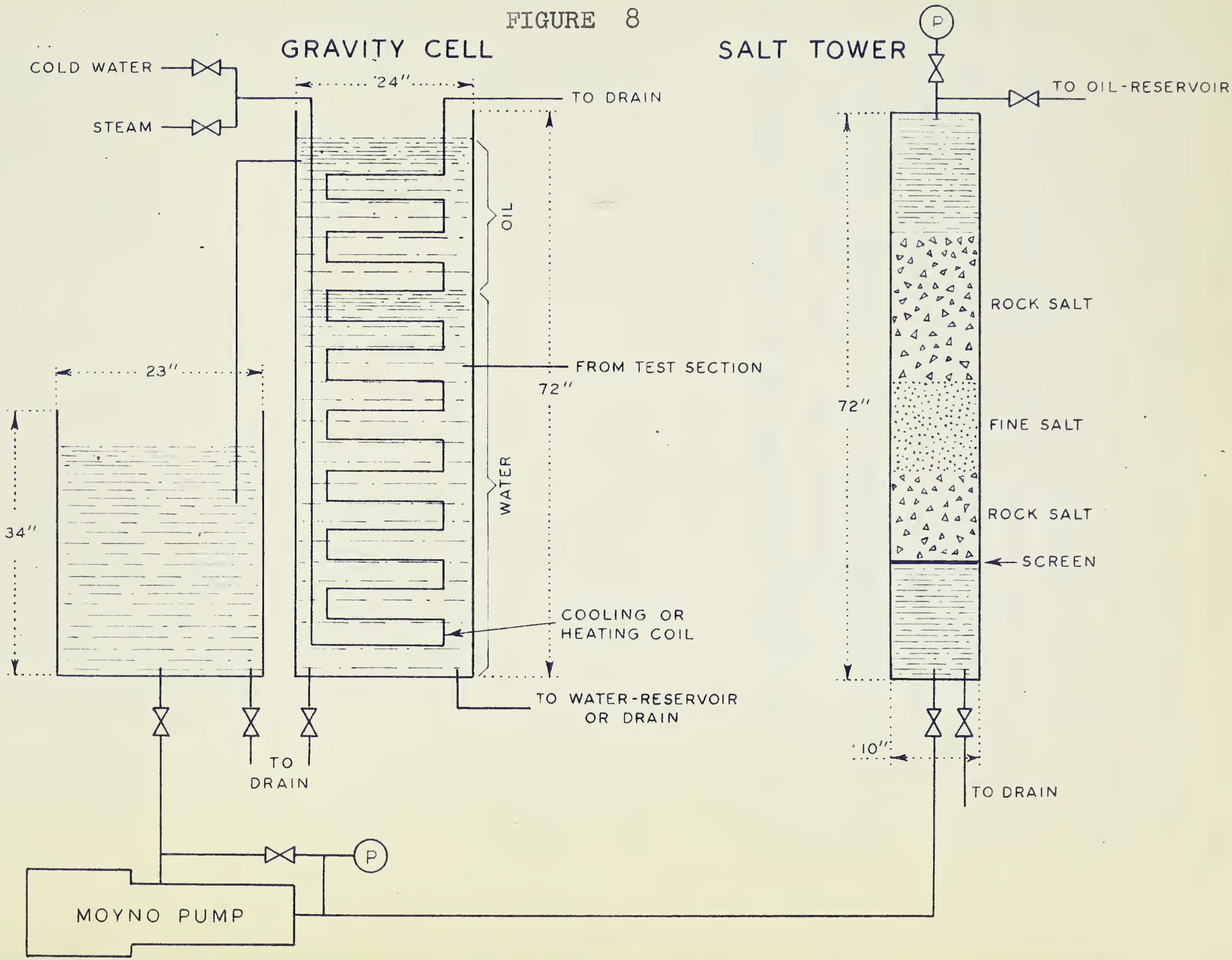
The pressure measuring system was checked by making a series of runs with water alone and clear oil alone. The results of this calibration are shown in Appendix C.

Oil-Water Separator

The separator for recovering the oil from the flow line discharge was originally designed to operate continuously but such severe emulsion problems were encountered that it could only do so at low flow rates. The gravity cell equipped with sight glasses was constructed of sheet metal and is shown in Figure 8. The mixture flowing from the test pipe was introduced at the centre and allowed to separate by gravity. If the separation was incomplete the process was conducted batch-wise and a steam coil used to break the water-in-oil emulsion. Water was allowed to flow from the bottom of the separator by gravity and was either recycled back to the water reservoir or discharged into the sewer. The oil was decanted into the secondary drum where additional suspended water tended to settle out. It was then pumped through the salt tower to take out final traces of water. The salt tower was constructed of sheet metal. As shown in Figure 8 it was equipped inside with a supporting screen placed 1.5 feet from the bottom of the tower. A pressure

OIL WATER SEPARATOR

FIGURE 8



gage at the discharge end gave an indication of any blocking of the flow by salt. The return line from the salt tower to the oil reservoir was equipped with a salt trap and two screens to prevent salt from remaining in the test oil. The tower was packed from the bottom up with one foot of rock salt then one foot of fine salt and finally two feet of rock salt. Trapped water was drawn off at the drain. The through-put capacity of the tower was about 100 gallons of oil with 100 ppm water. The exhausted salt was not dissolved but appeared to be completely coated with the oil.

EXPERIMENTAL PROCEDURE

Pressure Differential Measurements

Differential pressure measurements were made with varying input ratios of oil to water from 0.1 to 10 for 13 different superficial water rates ranging from 0.116 ft per sec to 3.55 ft per sec. The measurements were made as follows:

(a) All manometers were vented and balanced and the valves between the splash pots and the pipe test section were closed. The air reduction valves were closed.

(b) All valves on the test section between the input manifold and the separator were opened. The discharge bypass valves on the pumps were opened, and the water and oil pumps started. The valves controlling the flow to the rotameters were then set to give the desired flow readings.

(c) After a sufficient time was allowed for the oil and water flow to reach a steady state--this was determined both visually and by pressure drop measurements to be less than about one minute for the rates under study--the valves on the measuring stations which spanned the test section under consideration were slowly opened. Care was taken to allow only oil to enter the splash pots. Fluid levels in both measuring stations were adjusted to the same elevation by regulating the air pressure in the splash pot. Valves connecting the pressure stations to the suitable manometer were then opened, the splash pot levels readjusted and the pressure differential recorded.

(d) After the reading was recorded all valves on the pressure measuring system were closed and the manometer rebalanced by venting. The oil flow rate was then changed and the sequence of operation repeated to obtain the new pressure reading for the new flow conditions.

Hold-Up Measurements

For measuring hold-up in the test section the rotameters were set at the required rates and the pipeline was allowed to reach a steady state. The quick-closing valves were then closed and the pumps shut off with the single switch. The contents of the line were collected from the pipe by injecting compressed air and elevating the pipe. It was found that almost 95% of the material could be recovered from the line. The contents of the pipe were collected in a four litre measuring cylinder. The oil and water separated on standing and the volume of each was observed and the hold-up calculated.

Examination of Flow Pattern

A transparent material was chosen for the pipe test section so that visual observations could be made of the flowing material. At low flow rates a quiet interface existed between the oil and water streams. At high flow rates the interface tended to break up. A photographic record of typical flow patterns was made using a 4 x 5 inch Linhoff press camera with a 90 mm lens and a high speed electronic flash which effectively "stopped" the flow in the pipe at all the flow rates used in the investigation. The single flash unit was mounted below

the pipe at right angles to the camera and a flash duration of 1/2000 sec was used.

A special shield to exclude outside lighting was constructed. This shield provided a light source from just above the test section and a black background for the pipe itself.

Operation of the Separator

The tendency of the clear oil to form stable emulsions with water was not great at low water flow rates i. e. 0.116 to 0.718 ft per sec. Under these conditions the separator could be operated continuously. The interface level in the separator was maintained in the range of one to two feet below the oil overflow point by adjusting the water discharge. In this manner the oil residence time in the separator was as great as possible. It was customary to check the separation by sampling and determining the water concentration of the oil accumulation in the secondary separator before pumping it back to the intake reservoir.

At high flow rates i. e. 1.08 to 3.55 ft per sec of water, the separator was operated batch-wise to cope with the emulsification problem. The discharge from the test section was collected in the empty separator during a series of experiments which were continued until the separator was full. The flow line was shut down and the emulsion in the separator was broken down by heating to about 140° F. That temperature was maintained for several hours with the steam coil heater. When the water content of the oil fell to about 100 ppm the

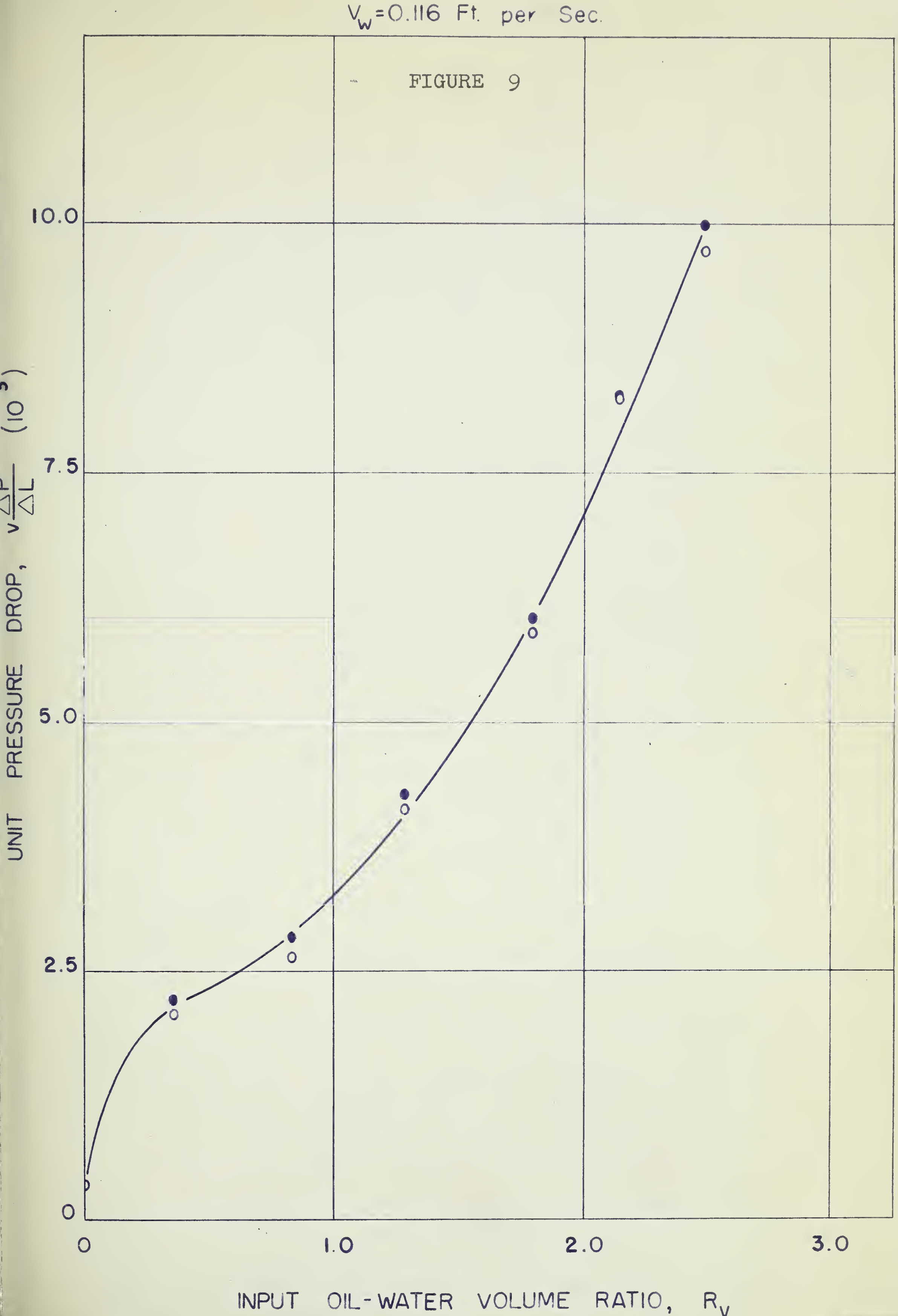
oil was pumped through the salt tower back to the intake reservoir. If the water content fell readily to less than 20 ppm in the separator the salt tower was not used. Although it was possible to tell the approximate water content by the cloudy or clear appearance of the oil, a Karl Fischer titration was generally used to check the water content of the final oil.

EXPERIMENTAL RESULTS AND DISCUSSION

Pressure drop data were obtained for 13 different water rates at input oil-water volume ratios varying from 0.1 to 10.0. Photographs were taken for the majority of water rates and ratios studied. Hold-up data at varying input ratios from 0.1 to 5 were obtained for all laminar water rates and for three different water rates in the turbulent region.

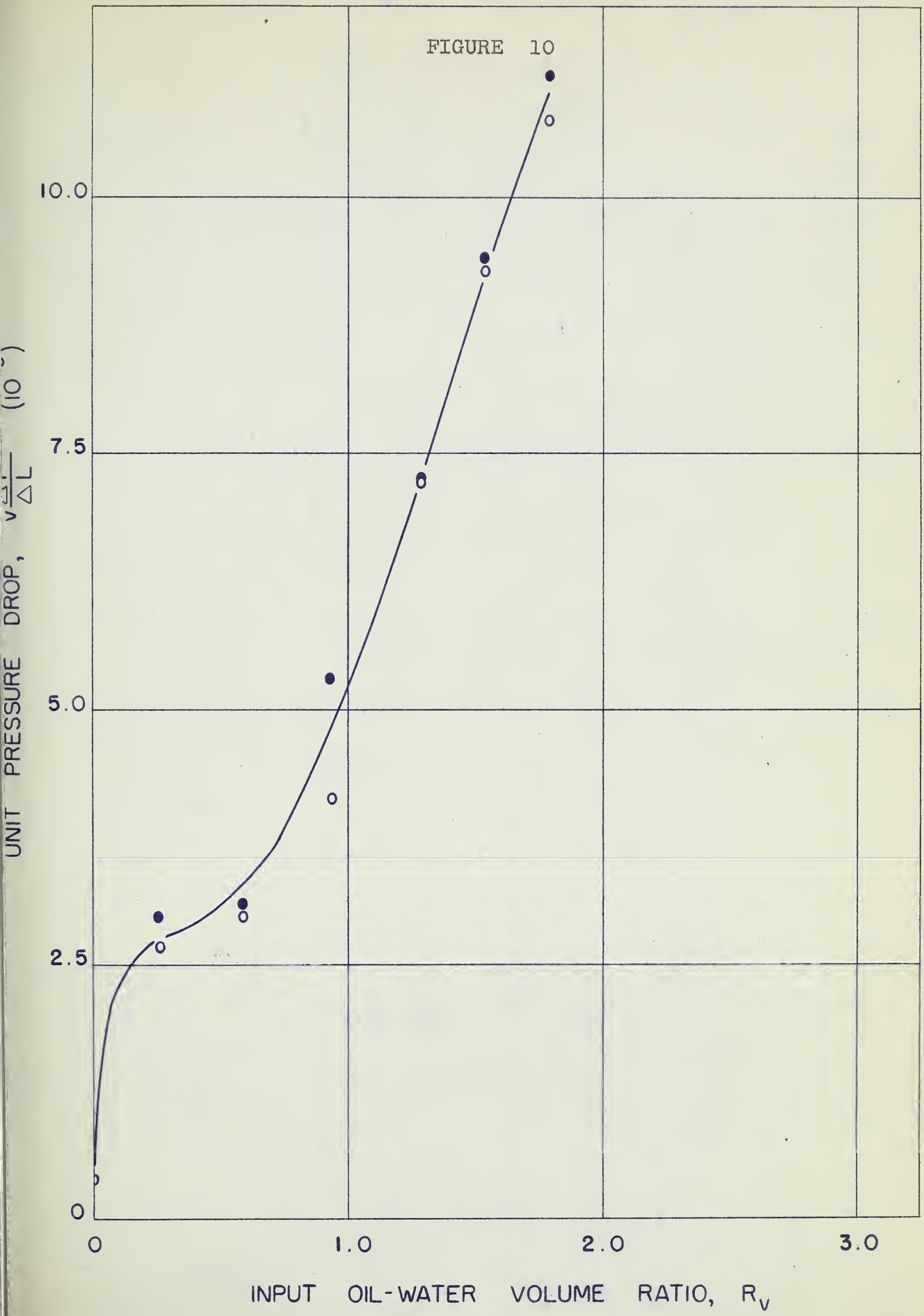
Pressure Drop

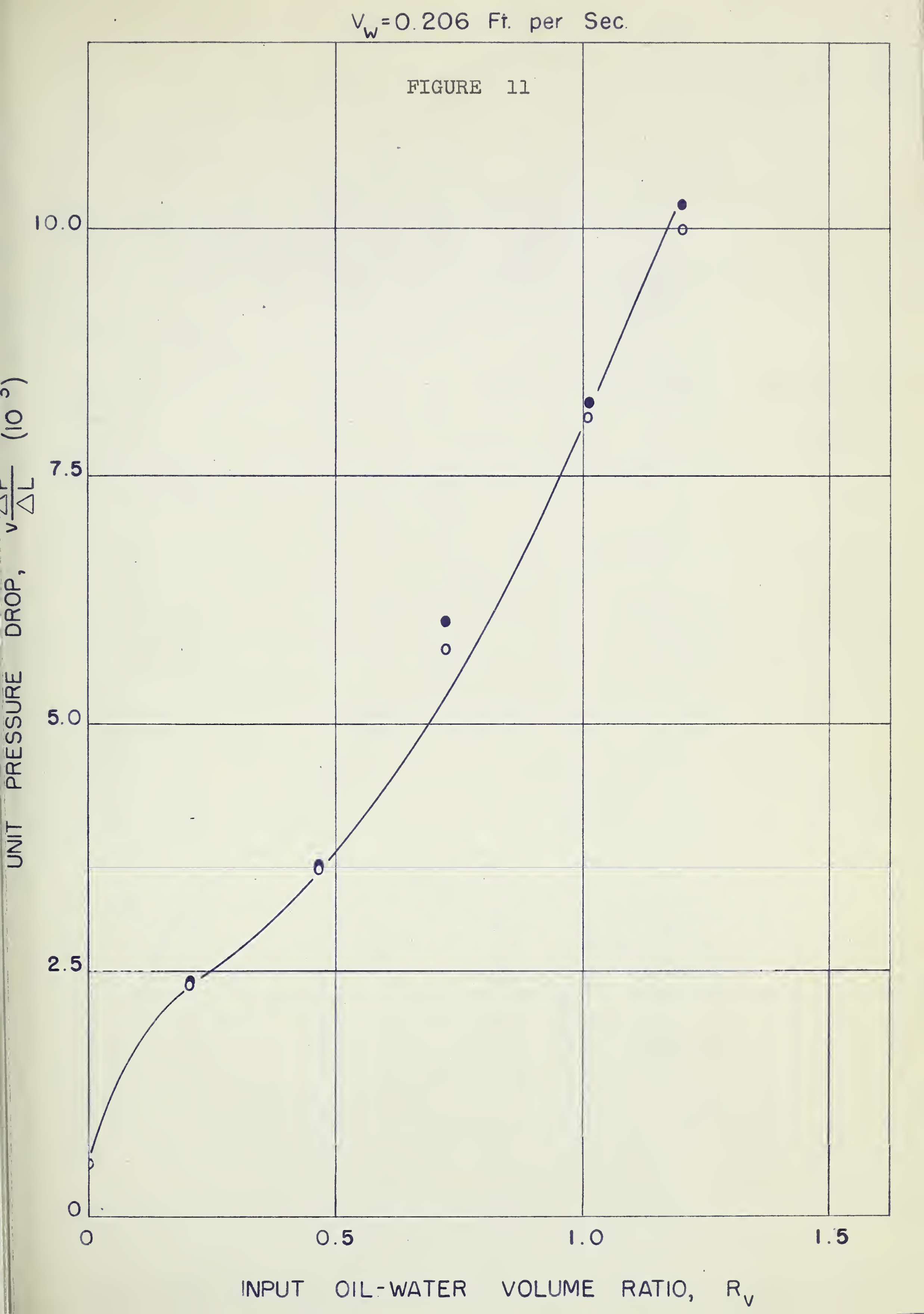
The pressure drop data are presented as unit pressure drop in feet of water per foot, $\frac{v\Delta P}{\Delta L}$, plotted against the input oil-water volume ratio, for each superficial water velocity in Figures 9 to 21. The superficial water velocity was calculated by dividing the water volumetric flow rate by the inside cross sectional area of the pipe. Figures 9 to 14 at the lower water rates show only readings measured over the complete test section. Measurements over both the complete section and the second half of the section are shown in Figures 15 to 21. The purpose of plotting both was to detect by pressure drop measurements any changes in the flowing material as it was pumped along the test line. If the shear forces at the wall of the pipe had changed the character of the flowing material to any marked extent the measurement of both pressure differentials would have detected the change. The unit pressure drop values for both the half and complete test section are in good agree-



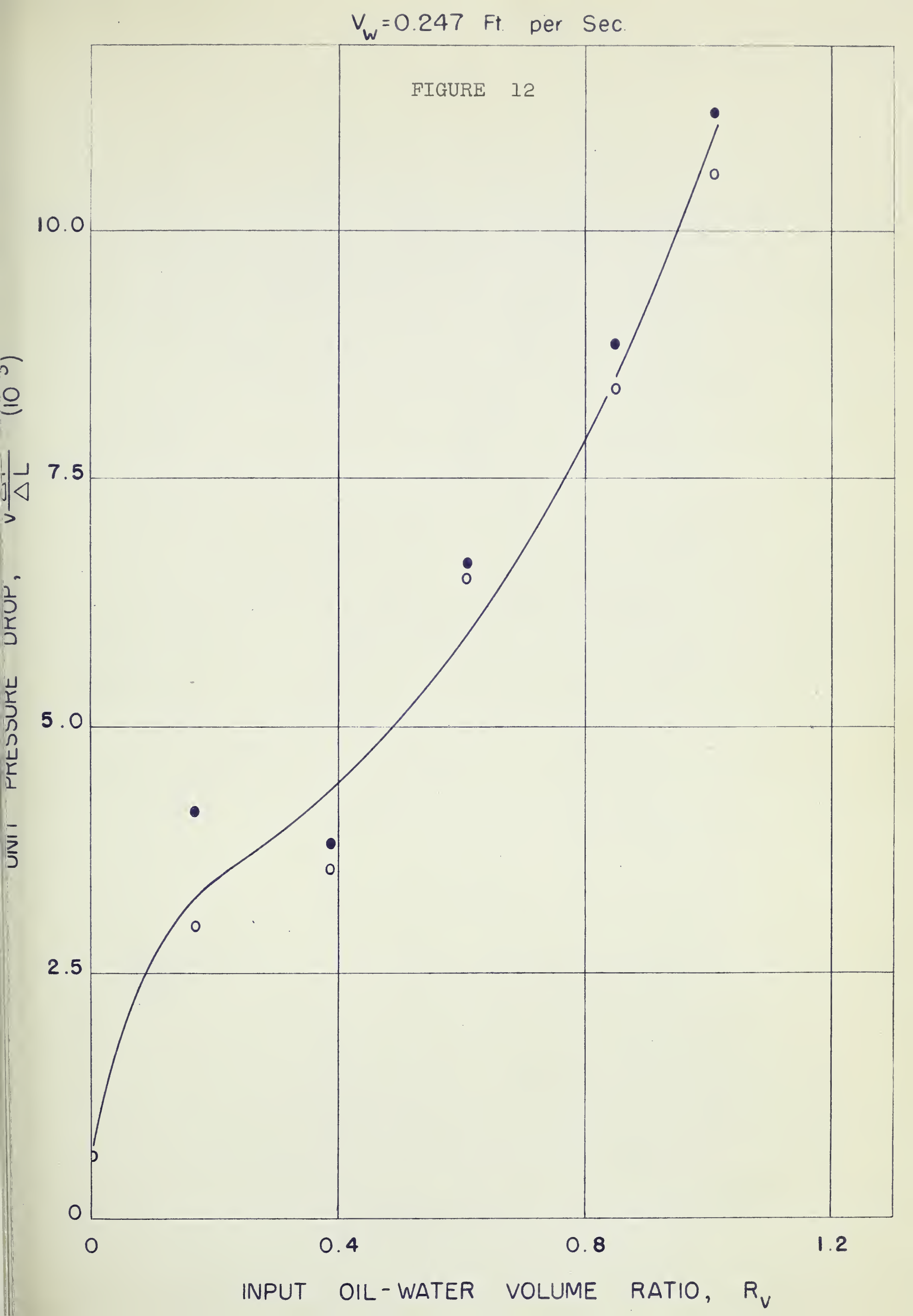
V_w=0.116 Ft. per Sec.
FIGURE 9
UNIT PRESSURE DROP, $v\frac{\Delta P}{\Delta L}$ (10^3)
10.0
7.5
5.0
2.5
0
0
1.0
2.0
3.0
INPUT OIL-WATER VOLUME RATIO, R_V

$V_w = 0.162$ Ft. per Sec.

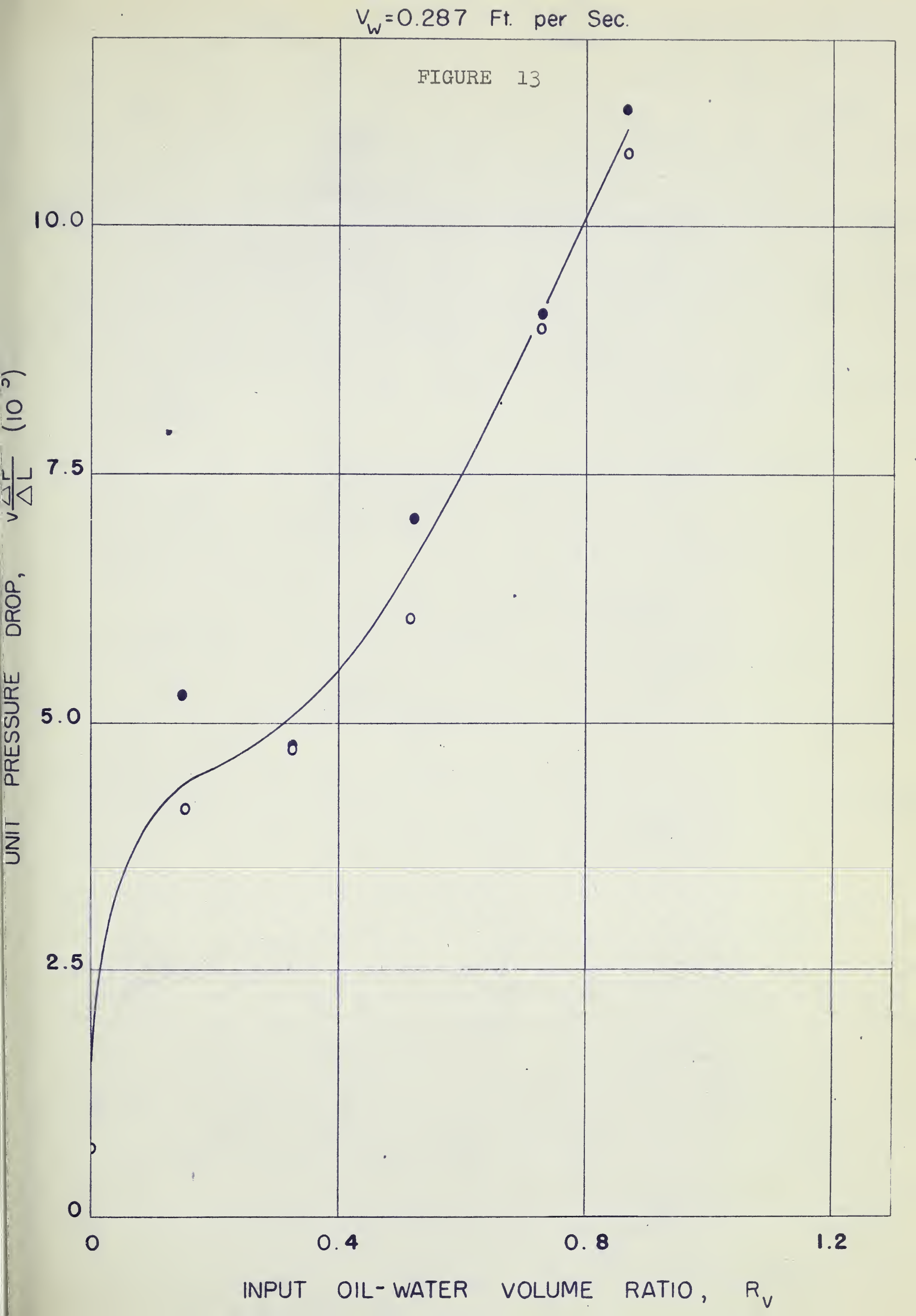




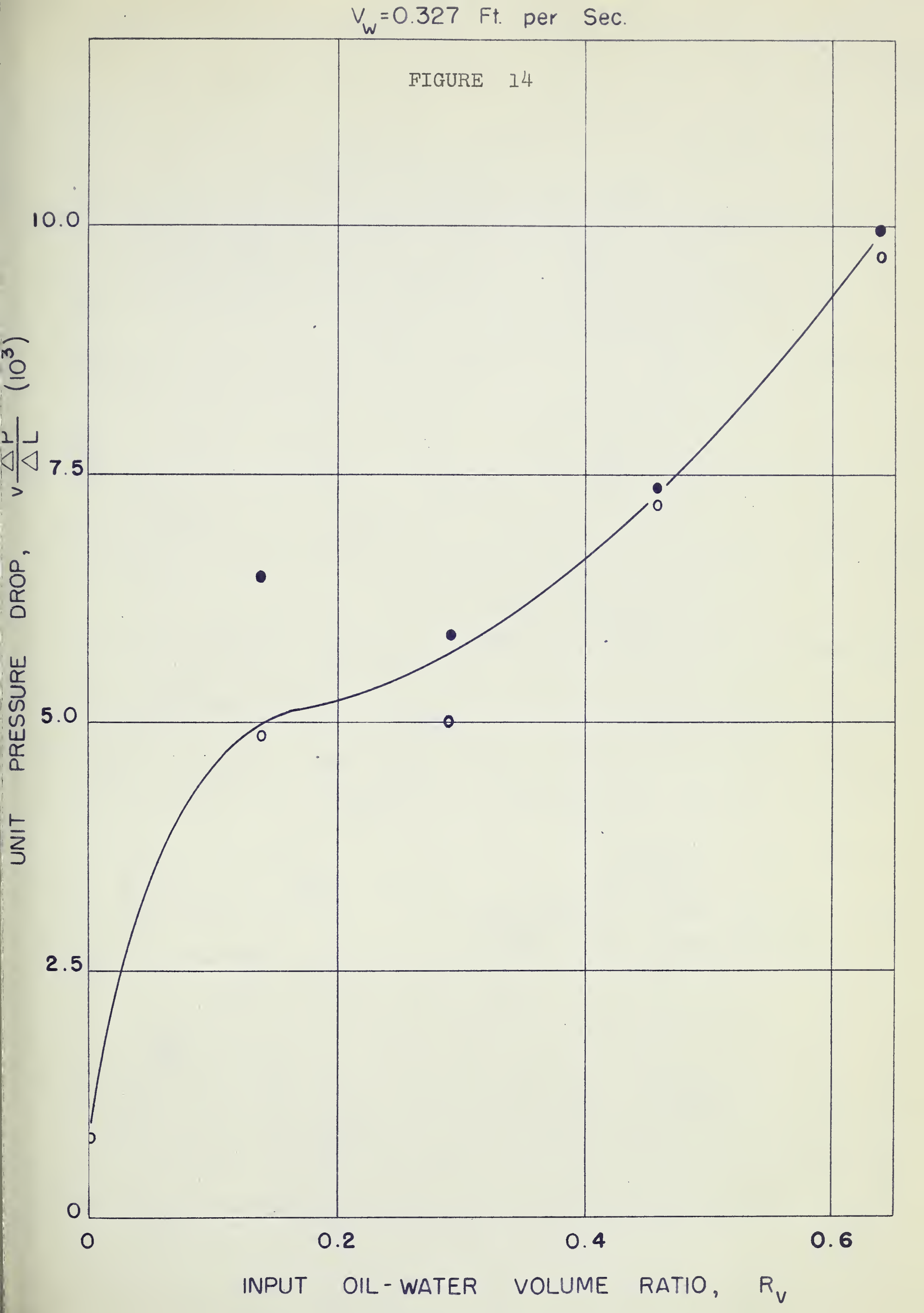
V_W = 0.206 Ft. per Sec.
FIGURE 11
10.0
7.5
5.0
2.5
0
0
0.5
1.0
1.5
UNIT PRESSURE DROP, $v\frac{\Delta P}{\Delta L}$ (10^3)
INPUT OIL-WATER VOLUME RATIO, R_V



V_w=0.247 Ft per Sec
FIGURE 12
10.0
7.5
5.0
2.5
0
0
0.4
0.8
1.2
INPUT OIL-WATER VOLUME RATIO, R_V

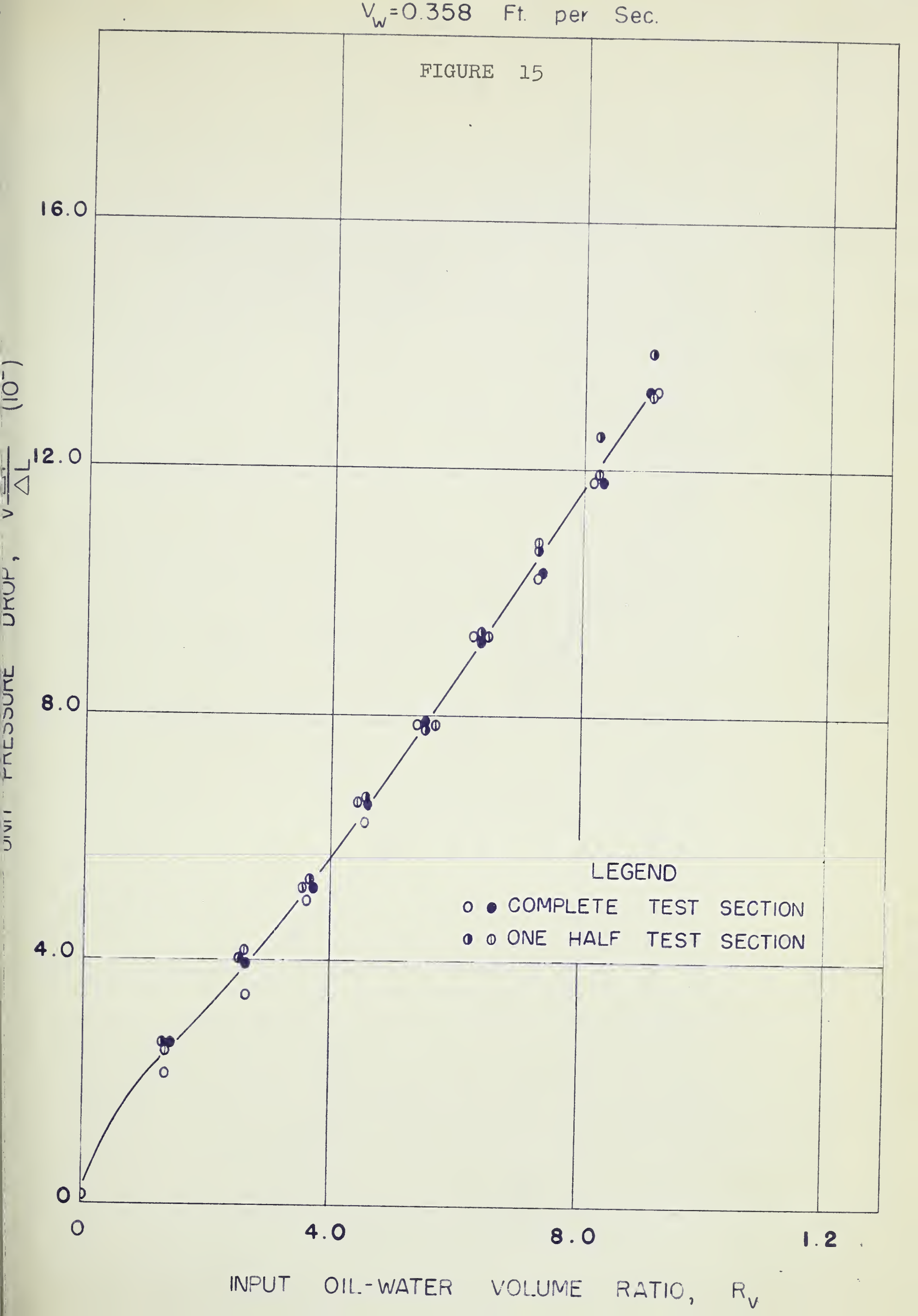


V_w=0.287 Ft. per Sec.
FIGURE 13
UNIT PRESSURE DROP, $\nu\frac{\Delta P}{\Delta L}$ (10^3)
10.0
7.5
5.0
2.5
0
0
0.4
0.8
1.2
INPUT OIL-WATER VOLUME RATIO, R_v

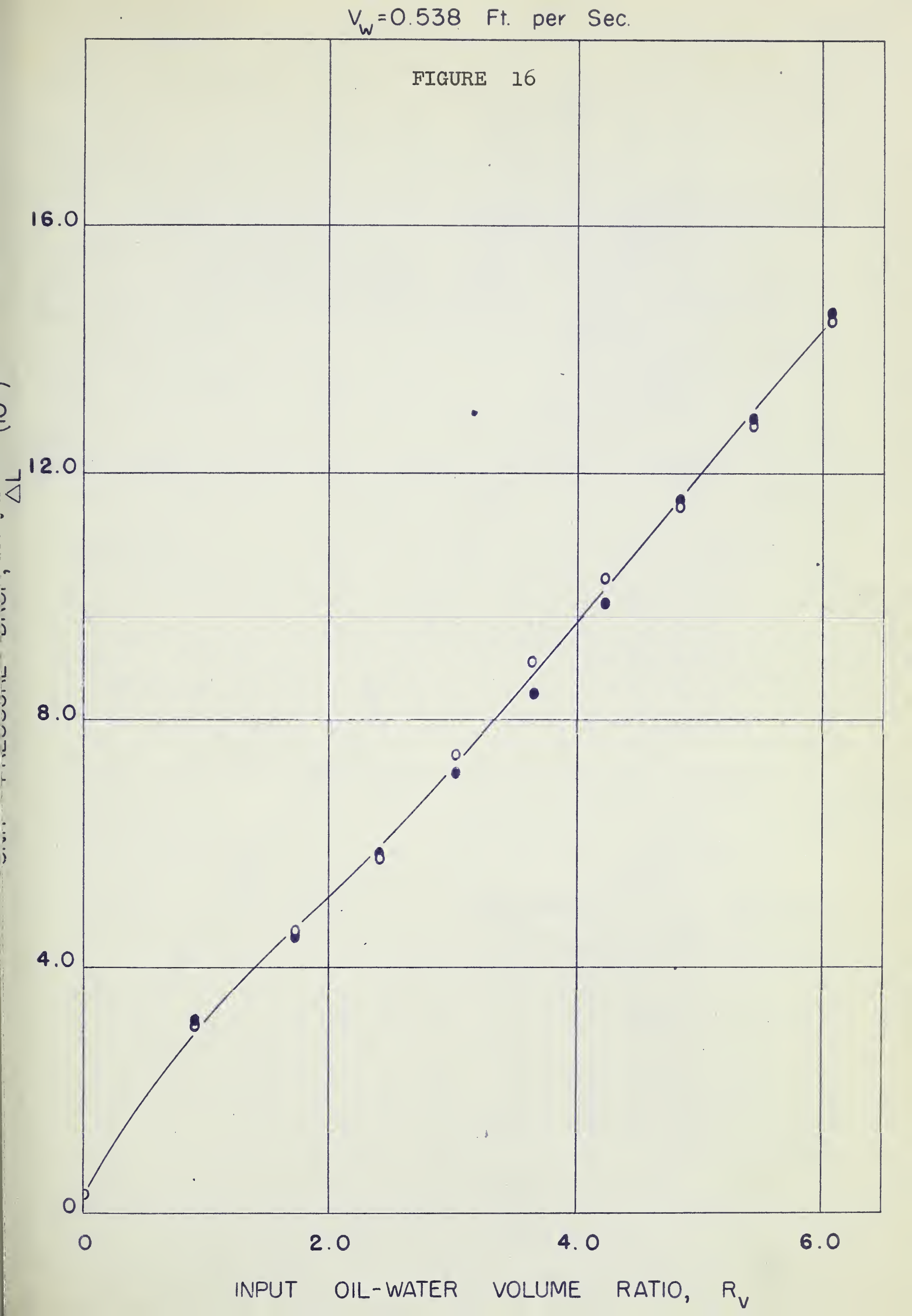


V_w=0.327 Ft. per Sec.
FIGURE 14
10.0
7.5
5.0
2.5
0
0
0.2
0.4
0.6
UNIT PRESSURE DROP, $v\frac{\Delta P}{\Delta L}$ (10^3)
INPUT OIL-WATER VOLUME RATIO, R_v

FIGURE 15



V_w=0.358 Ft. per Sec.
UNIT PRESSURE DROP, $V\frac{\Delta P}{\Delta L}$ (10^{-})
16.0
12.0
8.0
4.0
0
0
4.0
8.0
1.2
INPUT OIL-WATER VOLUME RATIO, R_V
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION



V_W = 0.538 Ft. per Sec.
FIGURE 16
16.0
12.0
8.0
4.0
0
0
2.0
4.0
6.0
INPUT OIL-WATER VOLUME RATIO, R_V
ΔL

$V_w = 0.718$ Ft. per Sec.

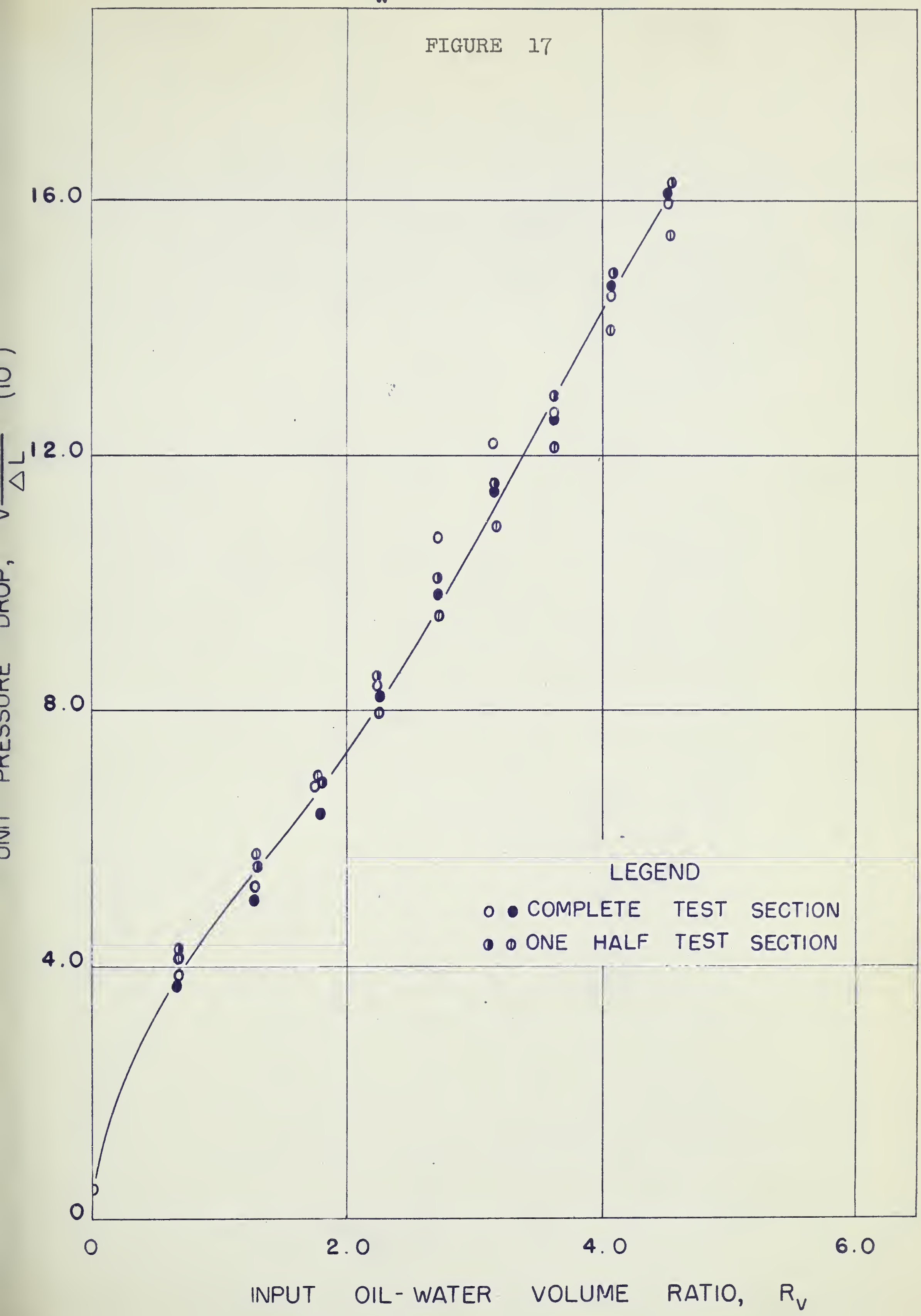
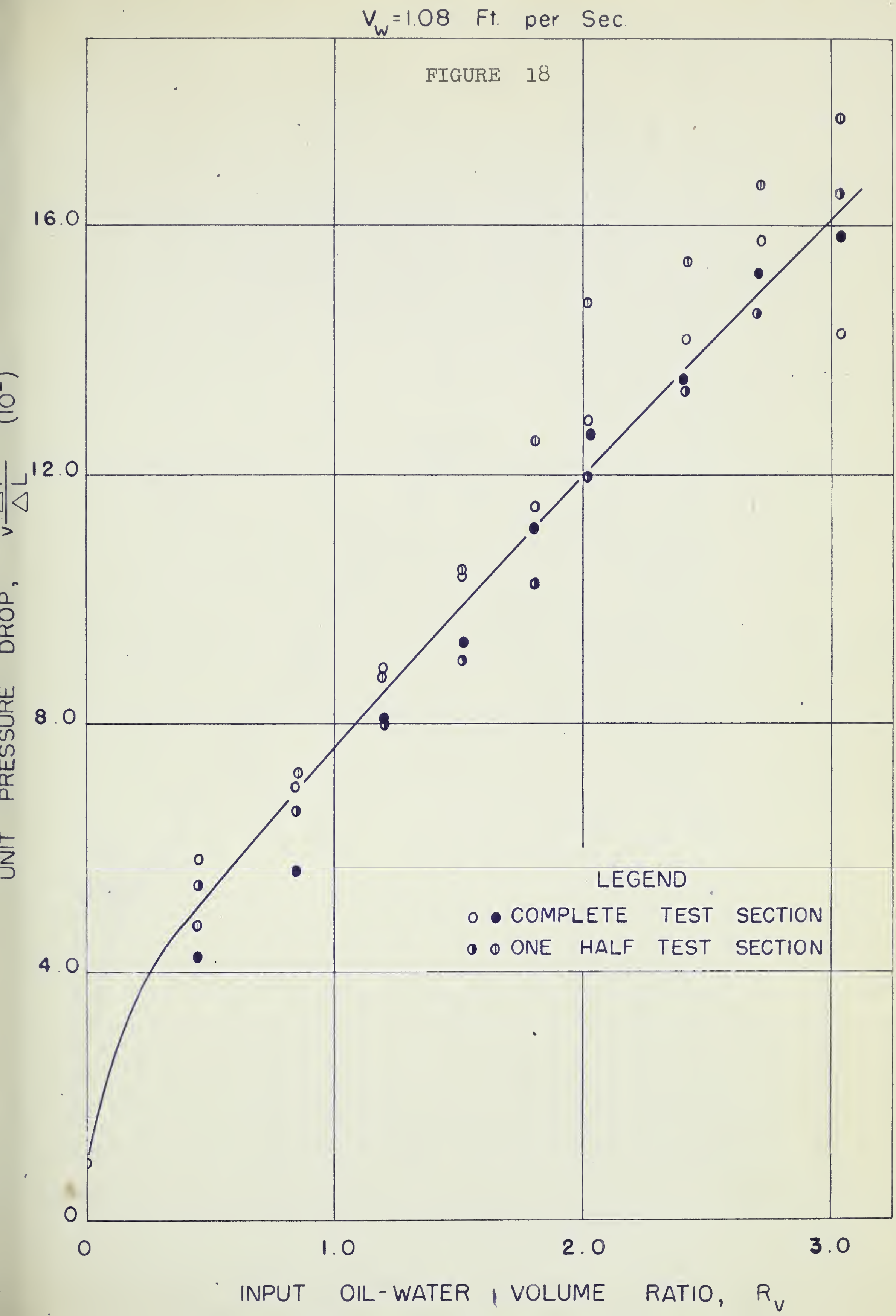
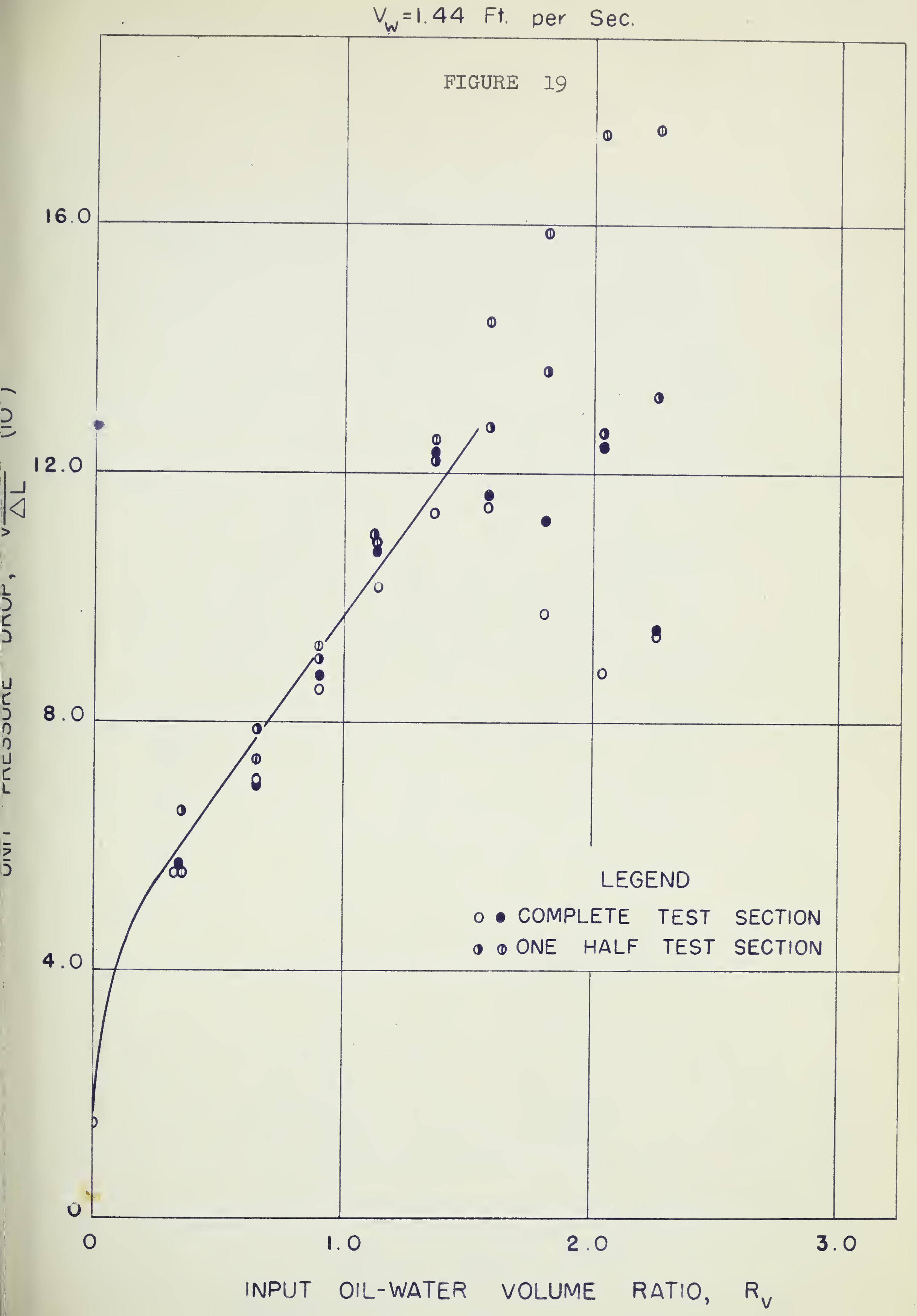


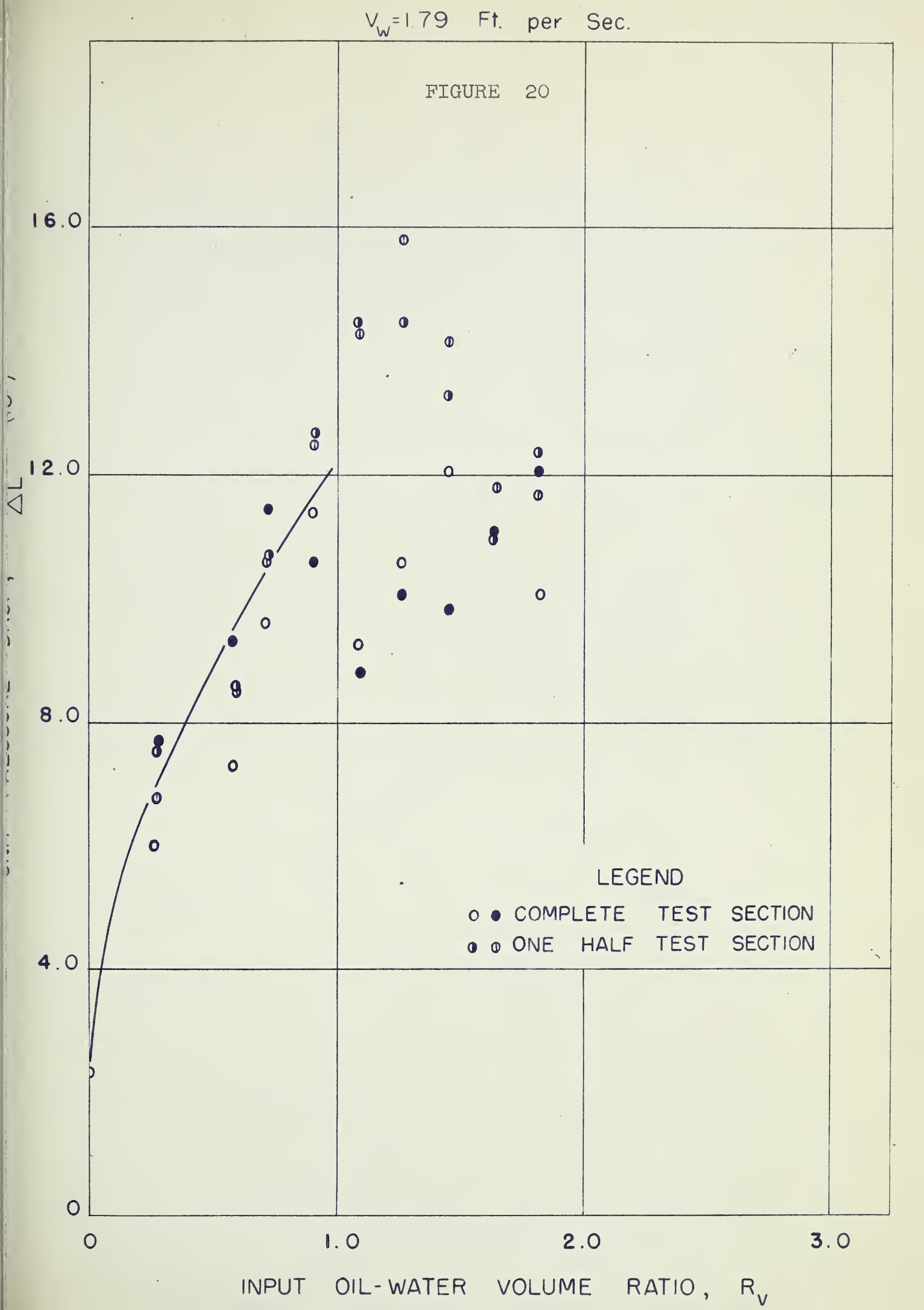
FIGURE 17



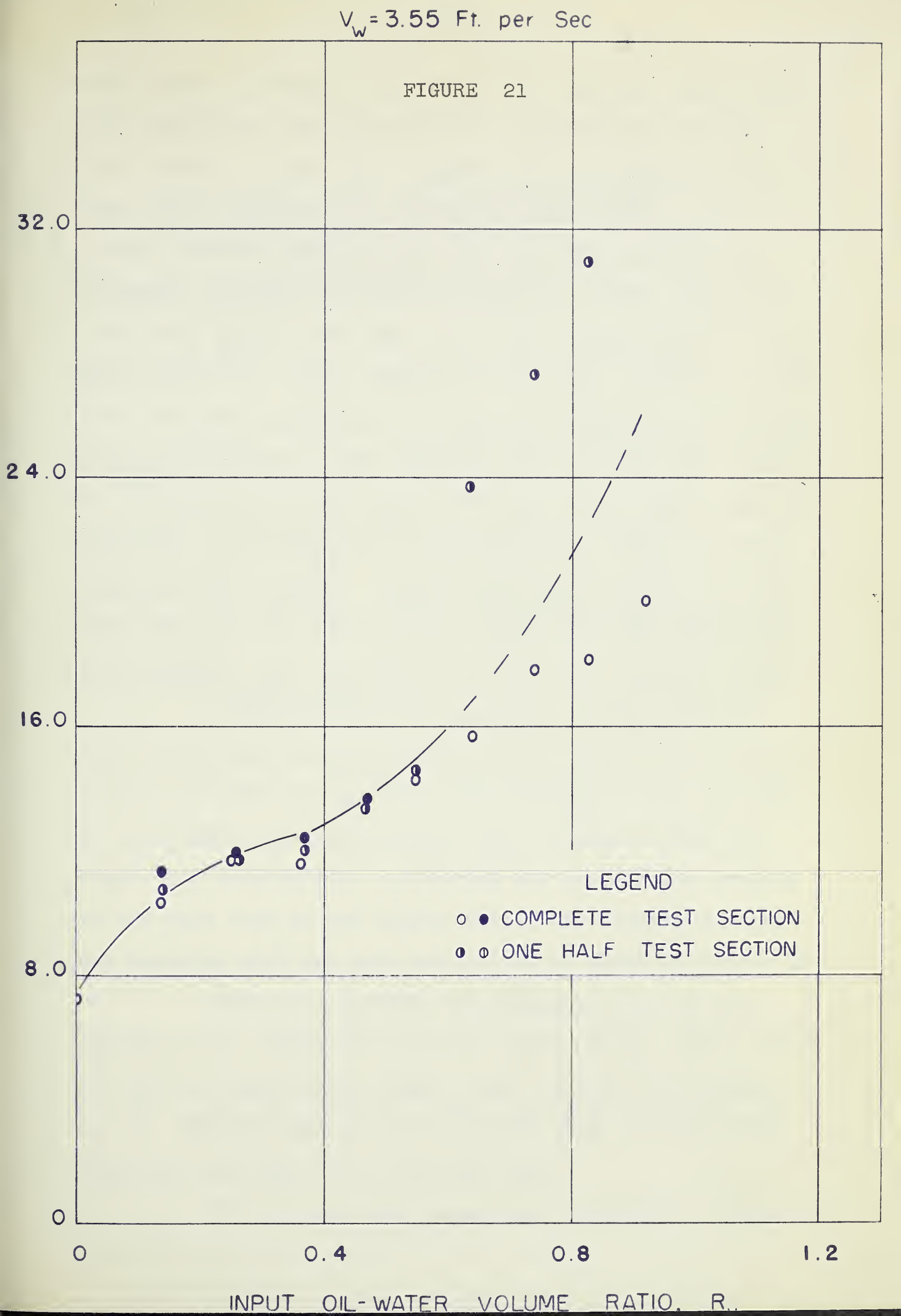
V_w = 1.08 Ft. per Sec.
FIGURE 18
16.0
12.0
8.0
4.0
0
1.0
2.0
3.0
UNIT PRESSURE DROP,
INPUT OIL-WATER VOLUME RATIO, R_V
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION



V_W = 1.44 Ft. per Sec.
FIGURE 19
16.0
12.0
8.0
4.0
0
1.0
2.0
3.0
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION
INPUT OIL-WATER VOLUME RATIO, R_V



V_w=1.79 Ft. per Sec.
FIGURE 20
16.0
12.0
8.0
4.0
0
ΔL
0
1.0
2.0
3.0
INPUT OIL-WATER VOLUME RATIO, R_V
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION


V_w = 3.55 Ft. per Sec
FIGURE 21
32.0
24.0
16.0
8.0
0
0
0.4
0.8
1.2
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION
INPUT OIL-WATER VOLUME RATIO, R.

ment except in Figures 19, 20 and 21. At these relatively high water rates the curves show a discontinuity at an input ratio of about 1.0. Figure 20 in particular shows that while complete test section results began to show a scatter from the smooth curve the half test section measurements appeared to continue along the curve. The discontinuity appeared to fade out at the higher ratio of 1.6. These curves are quite similar to the type of curve Sullivan (22) obtained by plotting the pressure drop due to friction alone in his work on the vertical flow of water and varsol. The unit pressure drop values are also of the same order of magnitude even though Sullivan employed a slightly larger diameter test section. Short's (18) work on air and water also indicated the same type of curve when the hydrostatic head component was separated from the unit pressure drop.

A discontinuity also appears in Figure 19 but since results are not available at the higher ratios it is not yet possible to report a disappearance of this scatter. At the higher superficial water rate of 3.55 ft per sec, Figure 21, considerable scattering was present due largely to the fact that at the higher ratios the flowing mixture was becoming more and more emulsified as shown in Figure 36.

There is a definite discontinuity in the unit pressure drop curves for the lower water rates. This occurs at very low input ratios and is most distinct in Figures 9 to 14. Similar observations have been made by Short (18) using air and water in a vertical pipe.

The pressure drop data were correlated in terms of

a modified Fanning type friction factor, f_w, based on the properties of the water and the superficial water velocity.

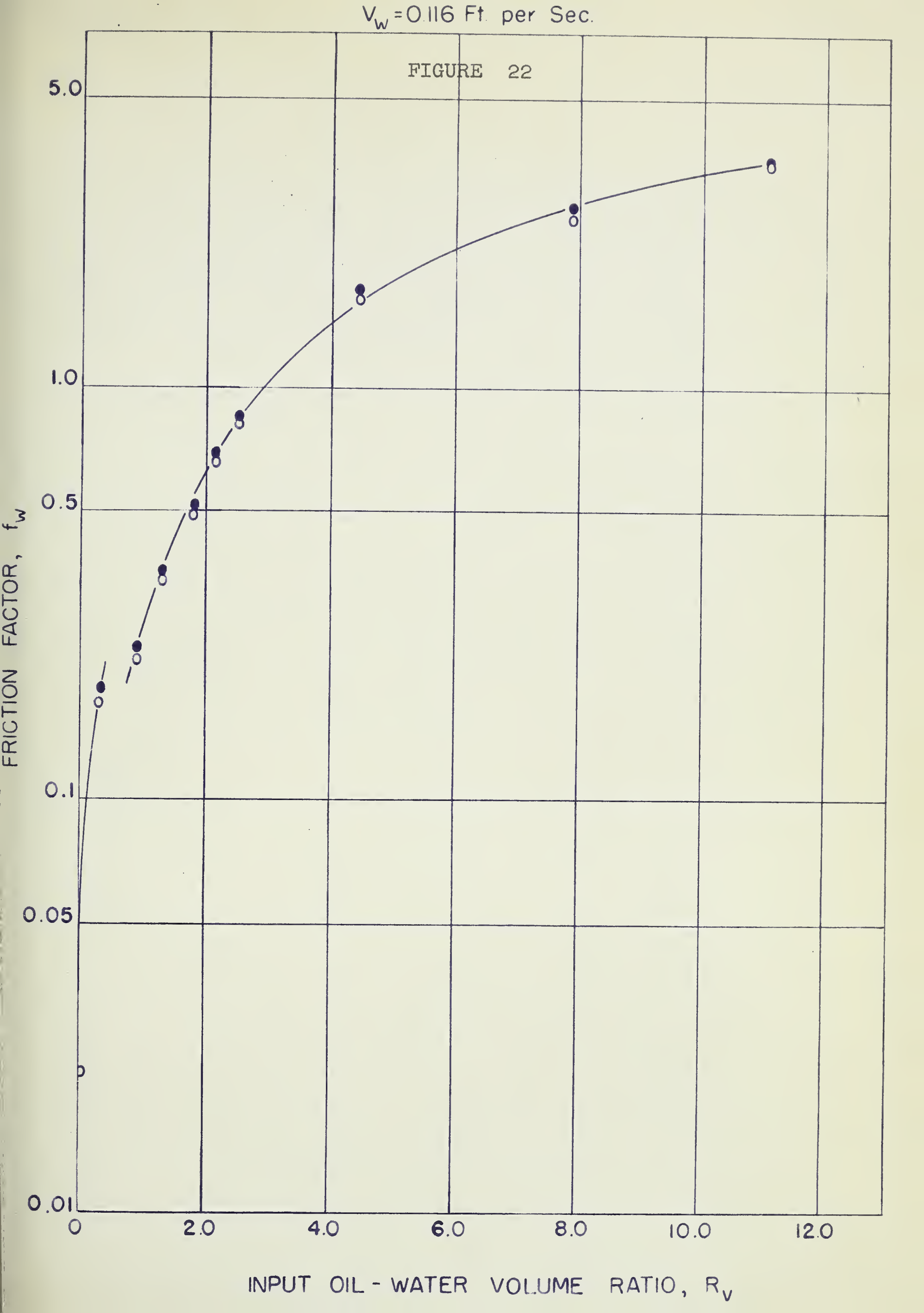
The modified friction factor is defined as

$$f_w = \frac{\Delta P \, g_c \, D}{\Delta L(2) V_w^2 \rho}$$

Curves of f_w versus input oil-water volume ratios at constant water velocity are shown in Figures 22 to 34, the calculations for which are in Appendix B. By interpolation from these curves, values of the friction factor were obtained for input oil-water volume ratios of 0.1, 0.2, 0.5, 1.0, 2.0, 5.0 and 10.0. These values are plotted on logarithmic graph paper against the superficial water velocity in Figure 35. In addition the f_w values for the water alone were plotted. Because the pipe was of a constant diameter and the properties of the water remained unchanged, the superficial water velocity V_w is directly proportional to a superficial Reynolds number for this series of experiments.

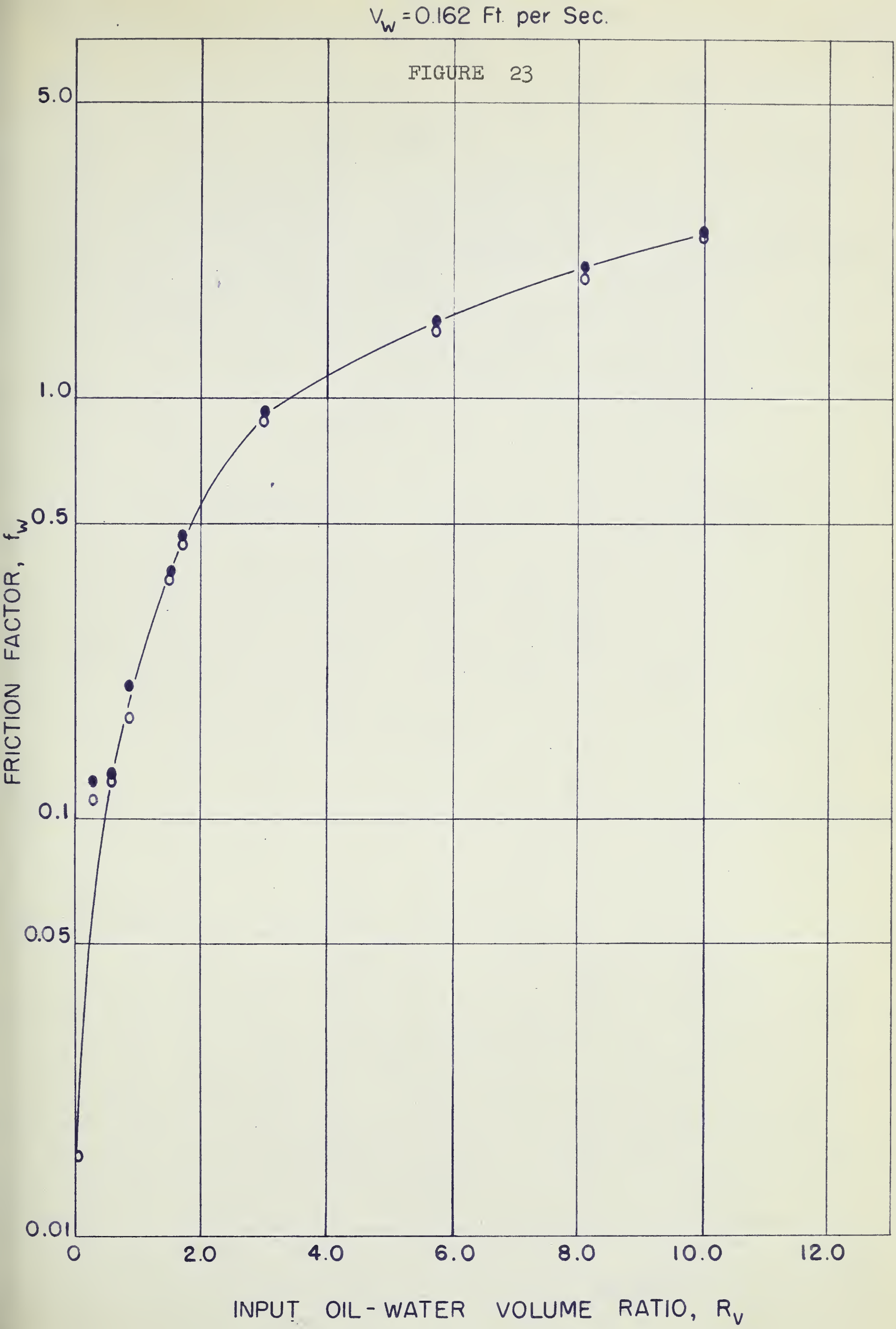
Because the superficial water velocity is proportional to a superficial Reynolds number, a picture of the liquid-liquid flow relative to the single phase correlation is obtained. The curves in Figure 35 form straight lines of slope equal to minus one in the laminar region, show the discontinuity present in the single phase line and then slope towards the single phase curve as the superficial water velocity increases.

A comparison of Figure 35, representing actual experimental data, with the results predicted in the theory

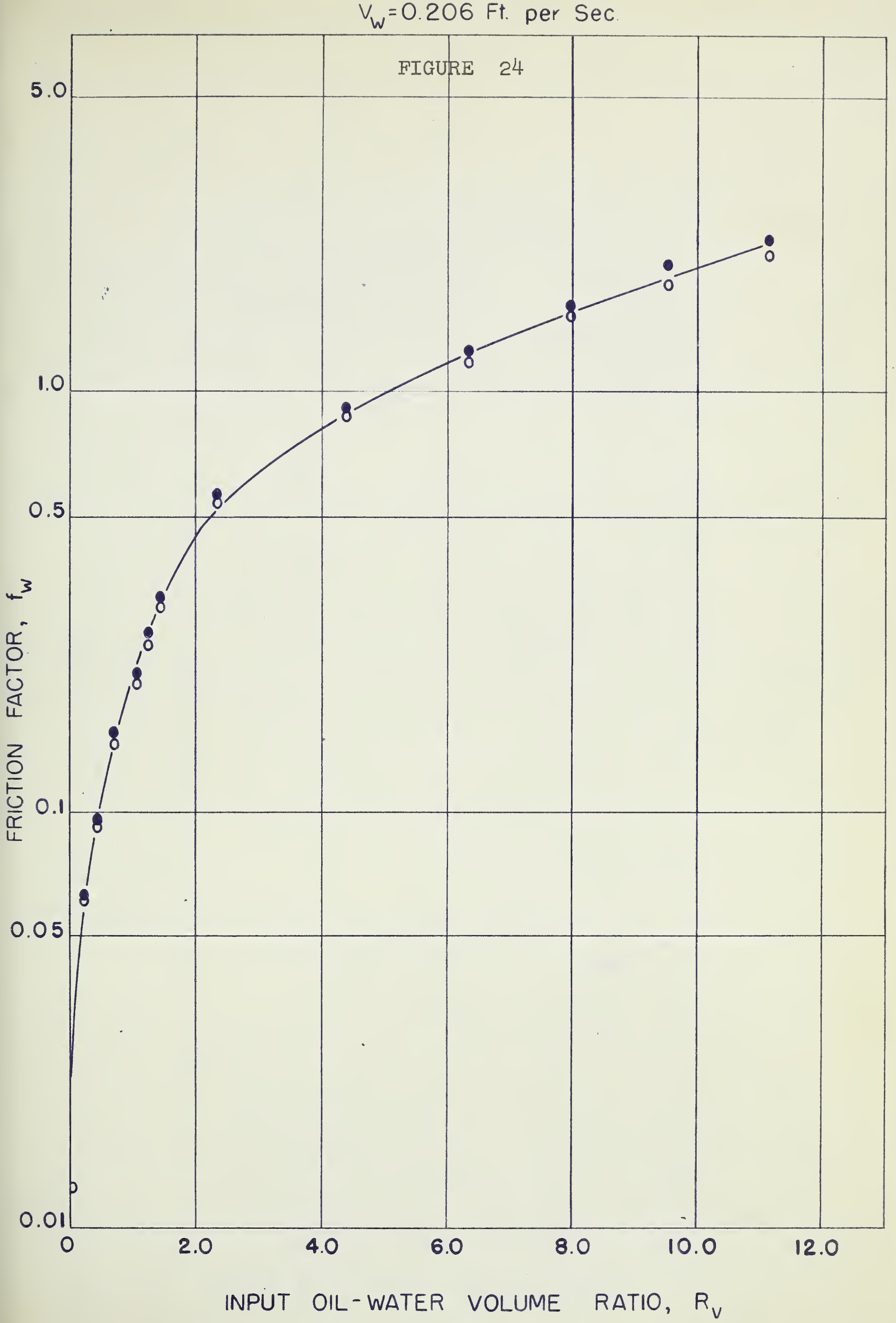


Vw = 0.116 Ft. per Sec.
FIGURE 22
5.0
1.0
0.5
0.1
0.05
0.01
FRICTION FACTOR, fw
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL - WATER VOLUME RATIO, Rv

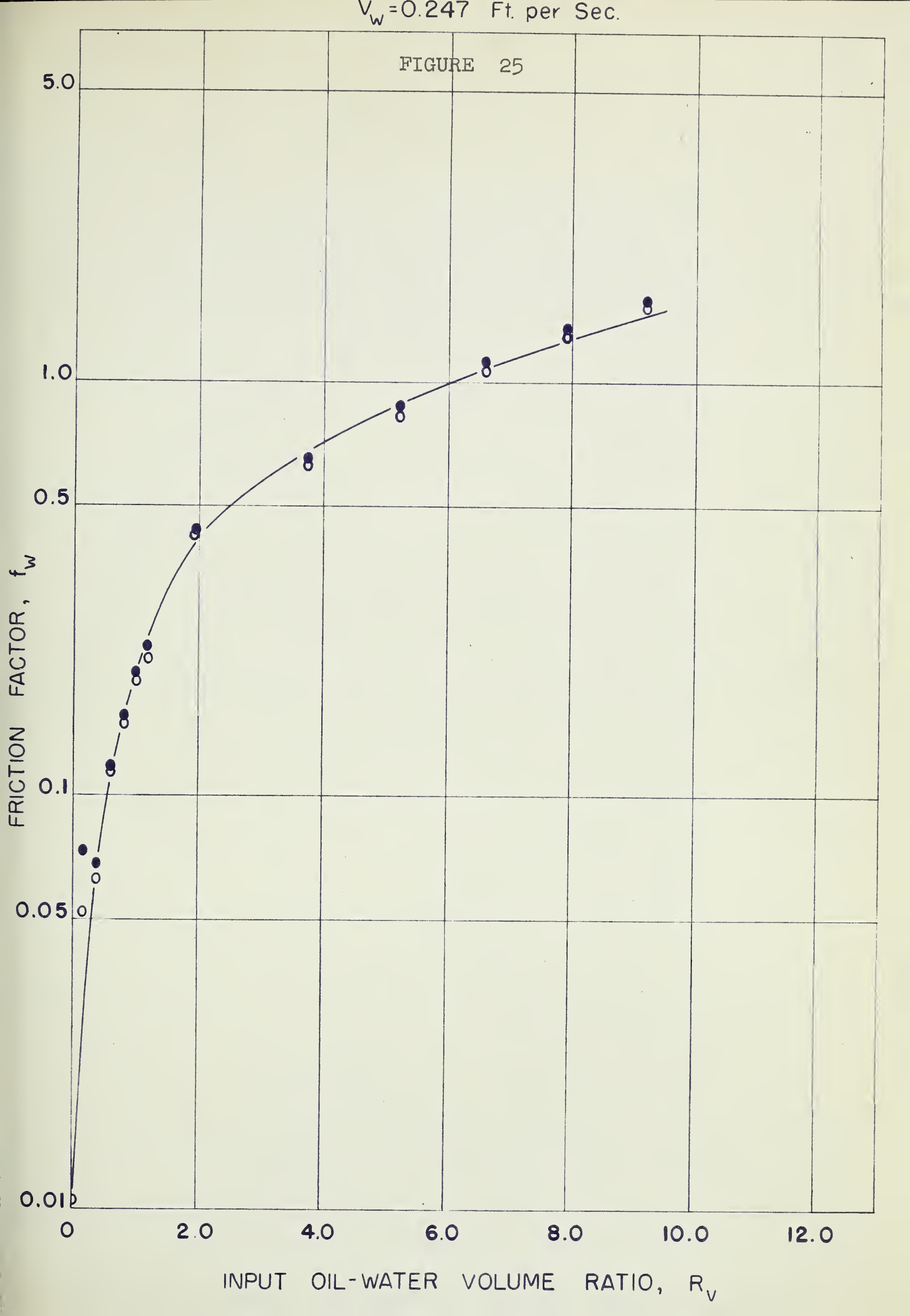
FIGURE 23



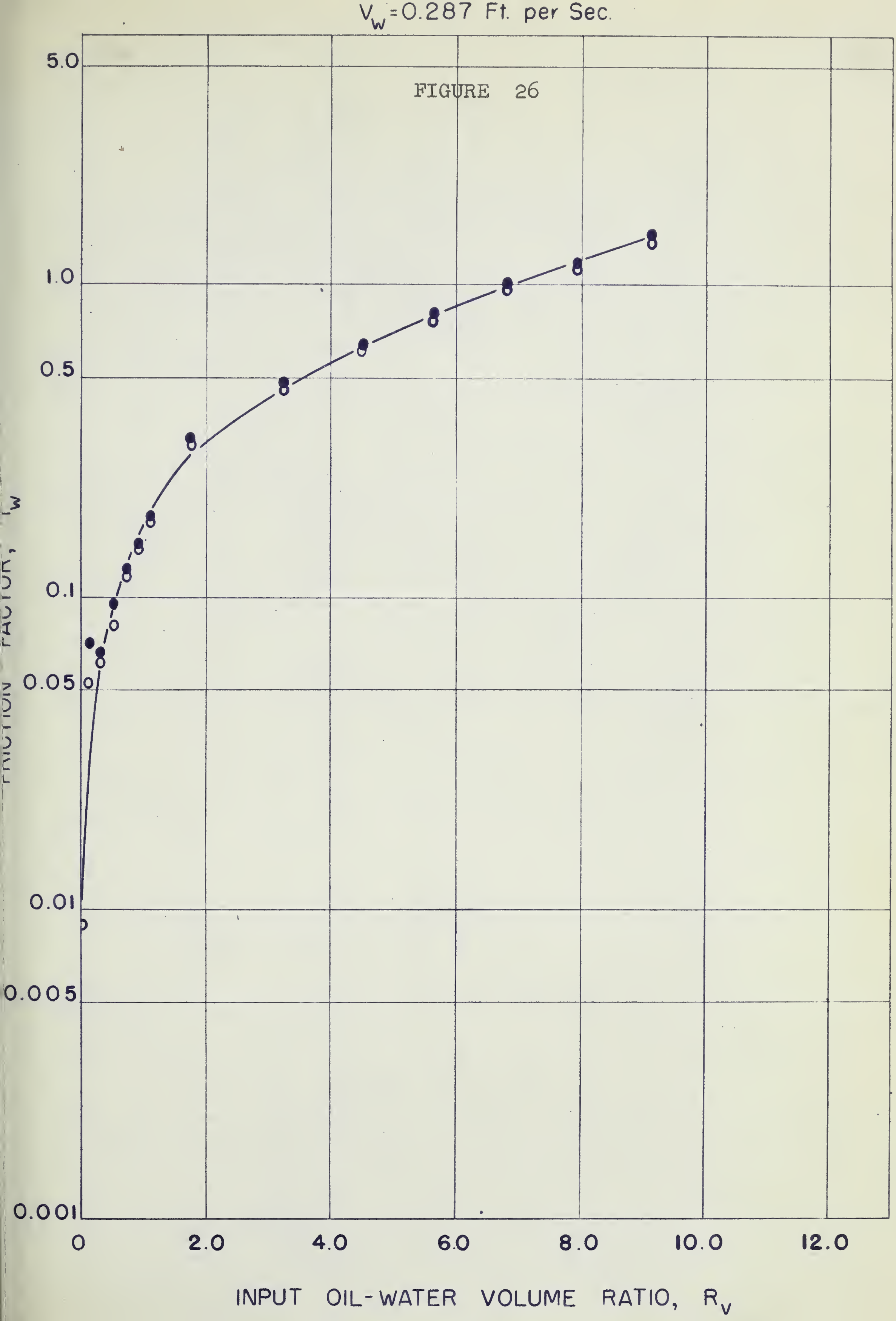
V_w = 0.162 Ft. per Sec.
5.0
1.0
0.5
0.1
0.05
0.01
FRICTION FACTOR, f_w
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL-WATER VOLUME RATIO, R_V

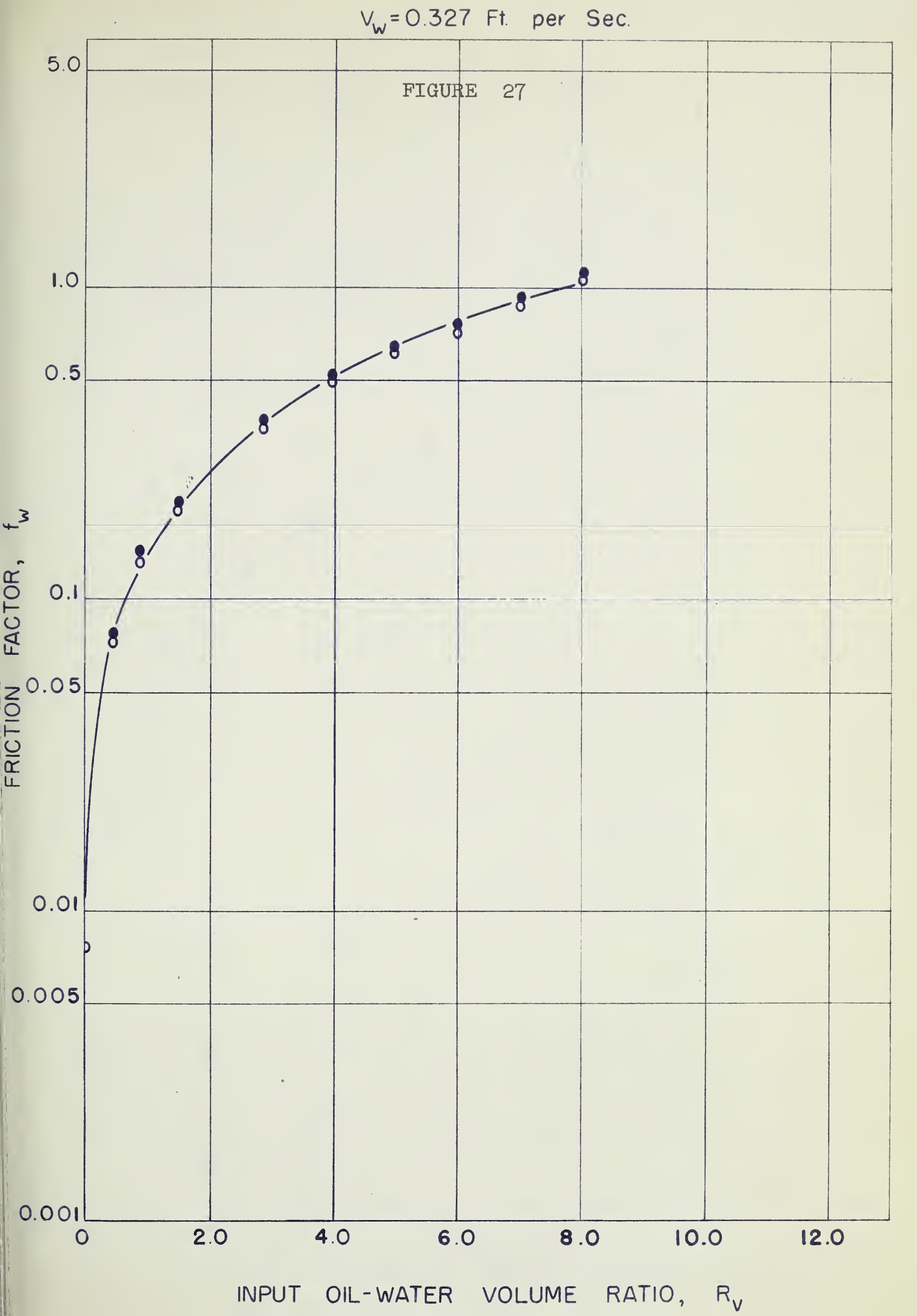


V_w=0.206 Ft. per Sec.
FIGURE 24
5.0
1.0
0.5
0.1
0.05
0.01
FRICTION FACTOR, f_w
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL-WATER VOLUME RATIO, R_V

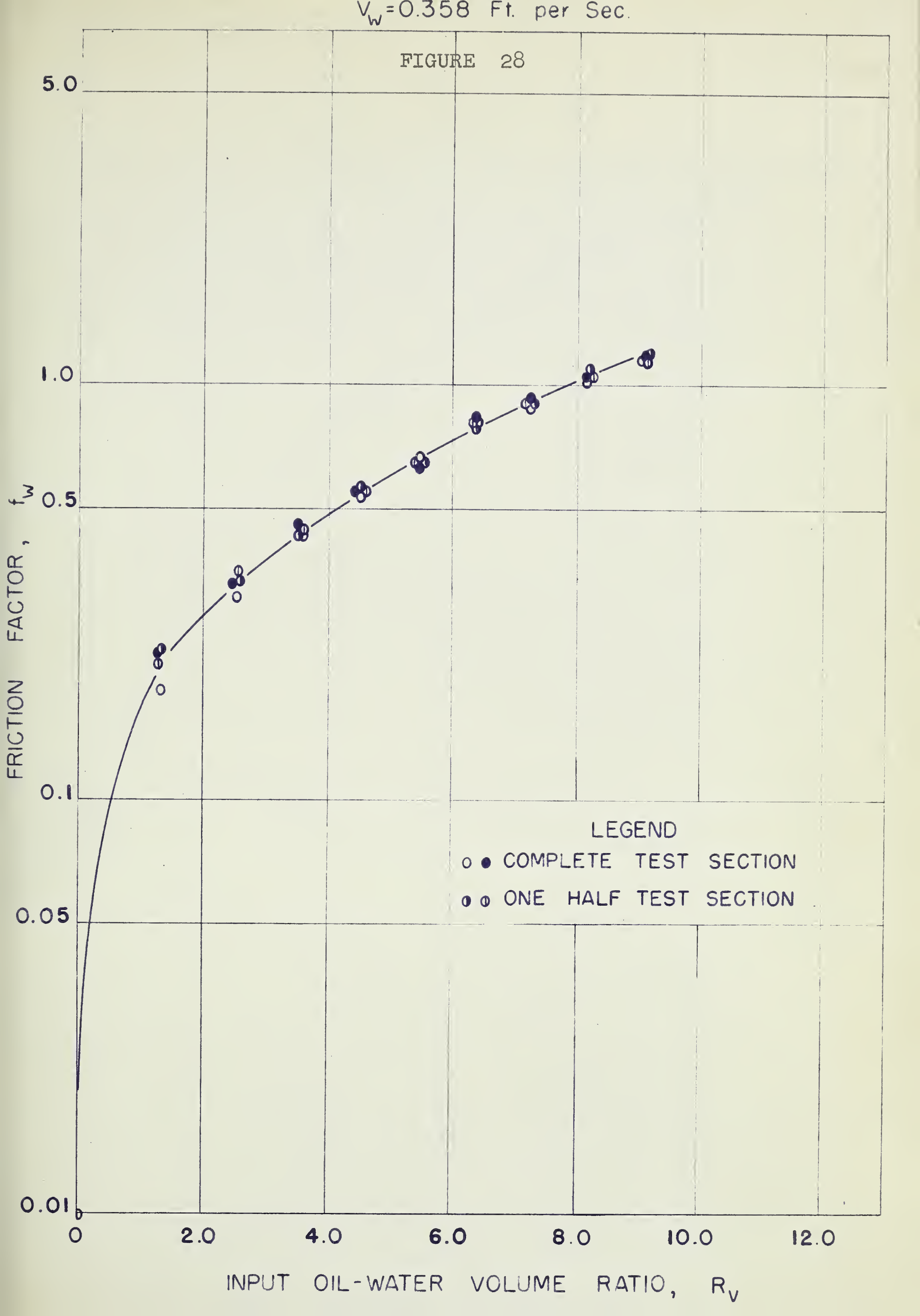


V_w = 0.247 Ft. per Sec.
FIGURE 25
5.0
1.0
0.5
0.1
0.05
0.01
FRICTION FACTOR, f_w
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL-WATER VOLUME RATIO, R_V

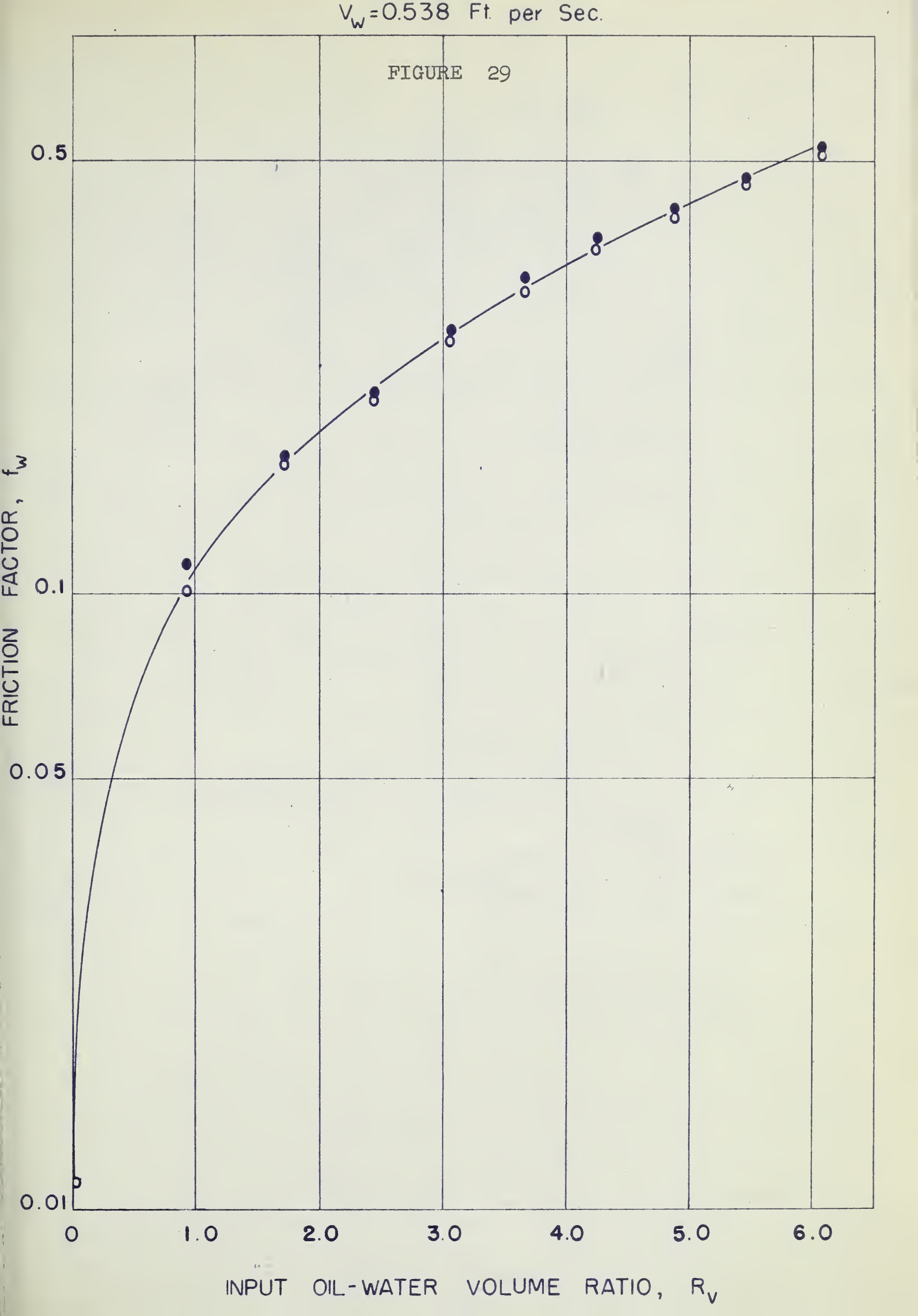

V_w=0.287 Ft. per Sec.
FIGURE 26
FRICTION FACTOR, f_w
5.0
1.0
0.5
0.1
0.05
0.01
0.005
0.001
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL-WATER VOLUME RATIO, R_v



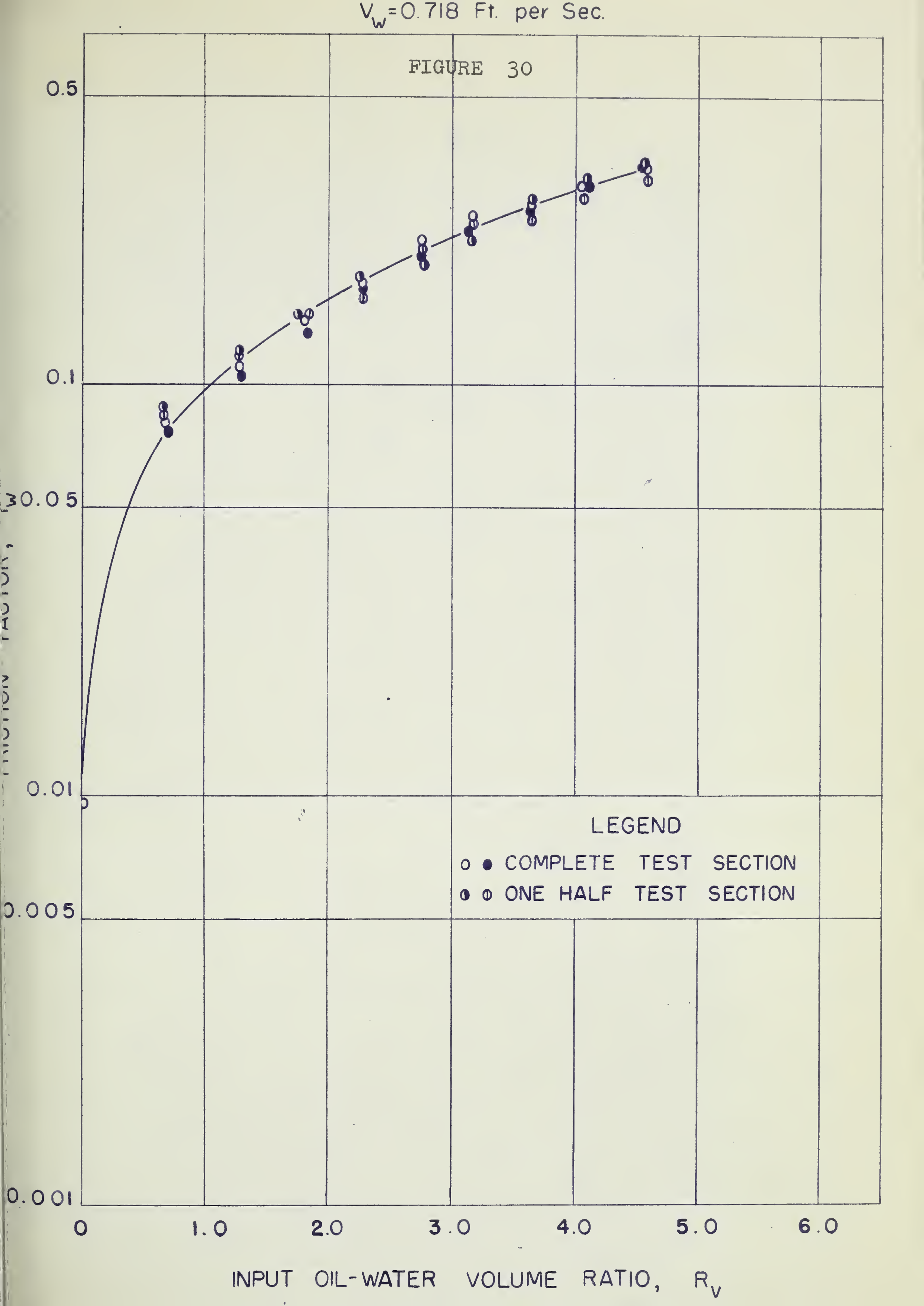
V_w = 0.327 Ft. per Sec.
FIGURE 27
5.0
1.0
0.5
0.1
0.05
0.01
0.005
0.001
FRICTION FACTOR, f_w
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL-WATER VOLUME RATIO, R_v



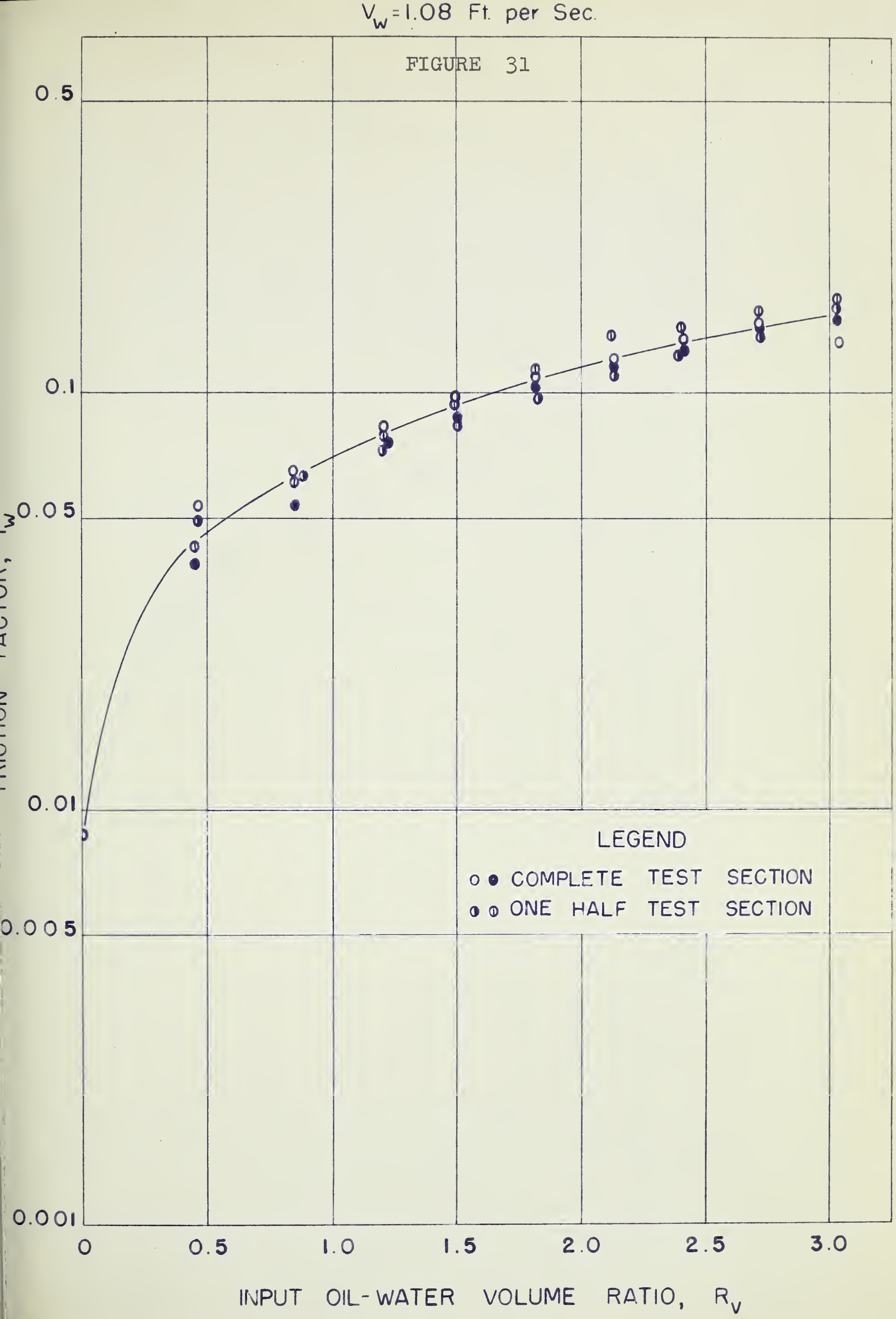
V_w=0.358 Ft. per Sec.
FIGURE 28
5.0
1.0
0.5
0.1
0.05
0.01
FRICTION FACTOR, f_w
0
2.0
4.0
6.0
8.0
10.0
12.0
INPUT OIL-WATER VOLUME RATIO, R_v
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION



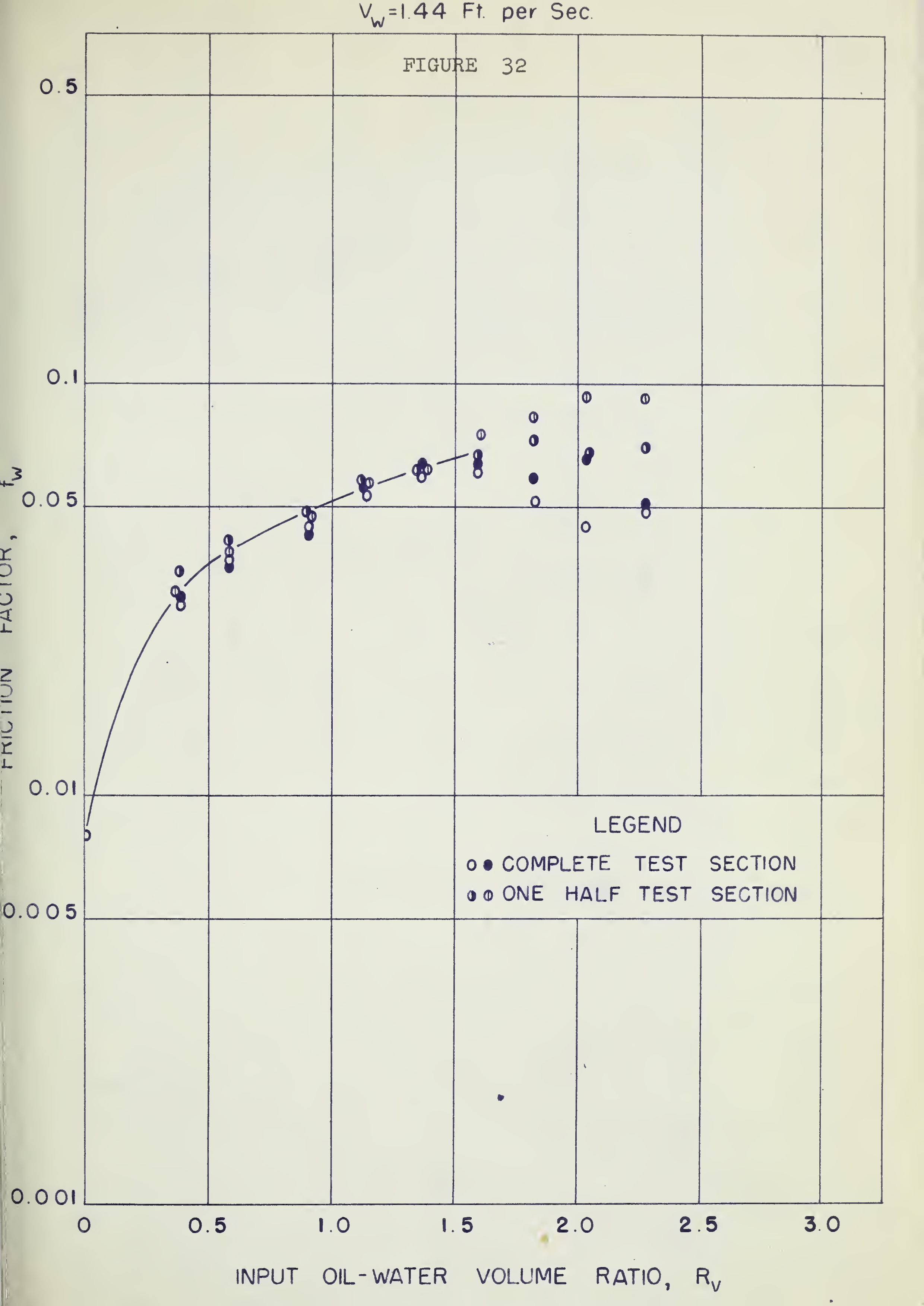
V_w = 0.538 Ft. per Sec.
FIGURE 29
FRICTION FACTOR, f_w
0.5
0.1
0.05
0.01
0
1.0
2.0
3.0
4.0
5.0
6.0
INPUT OIL-WATER VOLUME RATIO, R_v



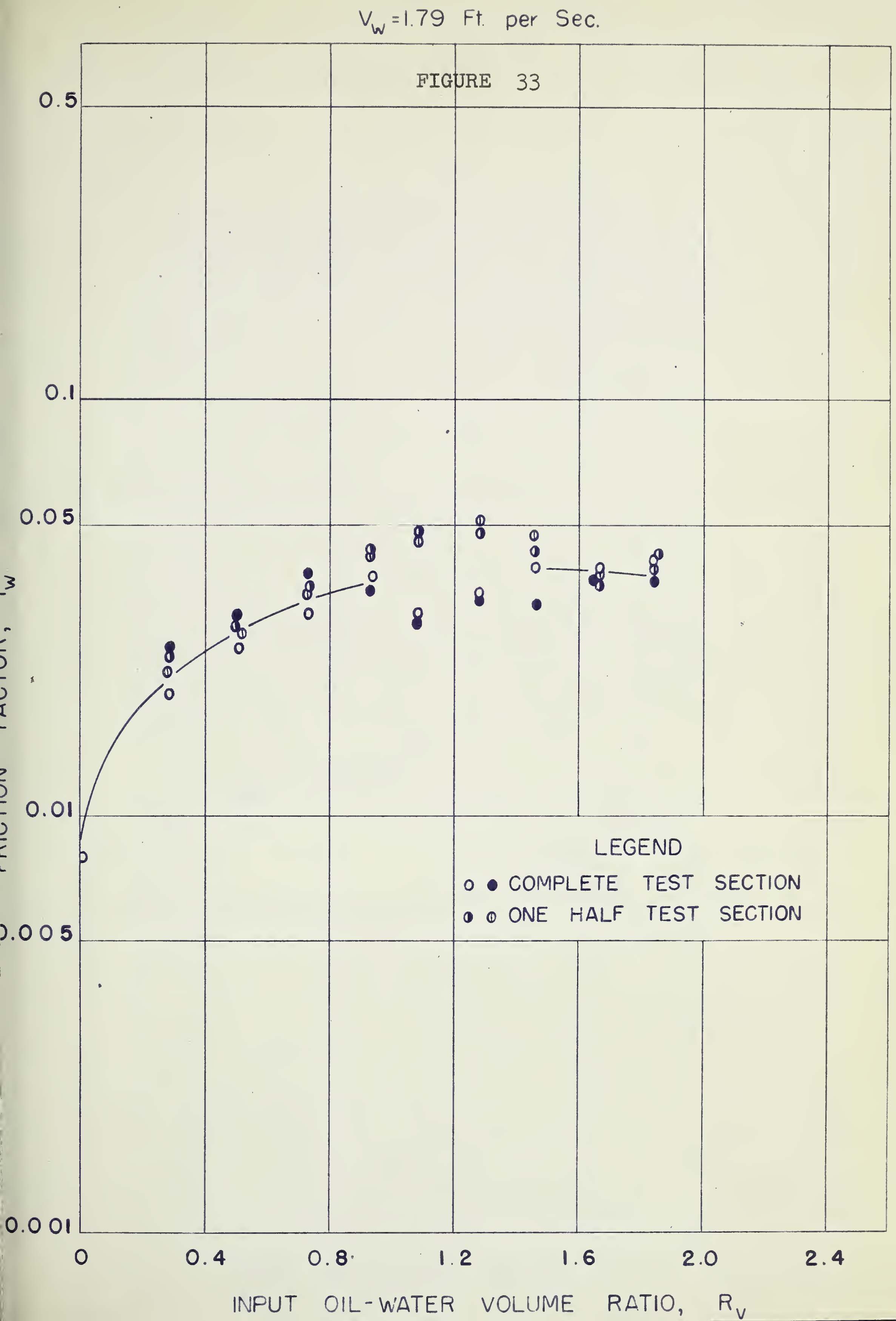
V_W=0.718 Ft. per Sec.
FIGURE 30
0.5
0.1
0.05
0.01
0.005
0.001
0
1.0
2.0
3.0
4.0
5.0
6.0
LEGEND
o ● COMPLETE TEST SECTION
◐ ⦶ ONE HALF TEST SECTION
INPUT OIL-WATER VOLUME RATIO, R_V

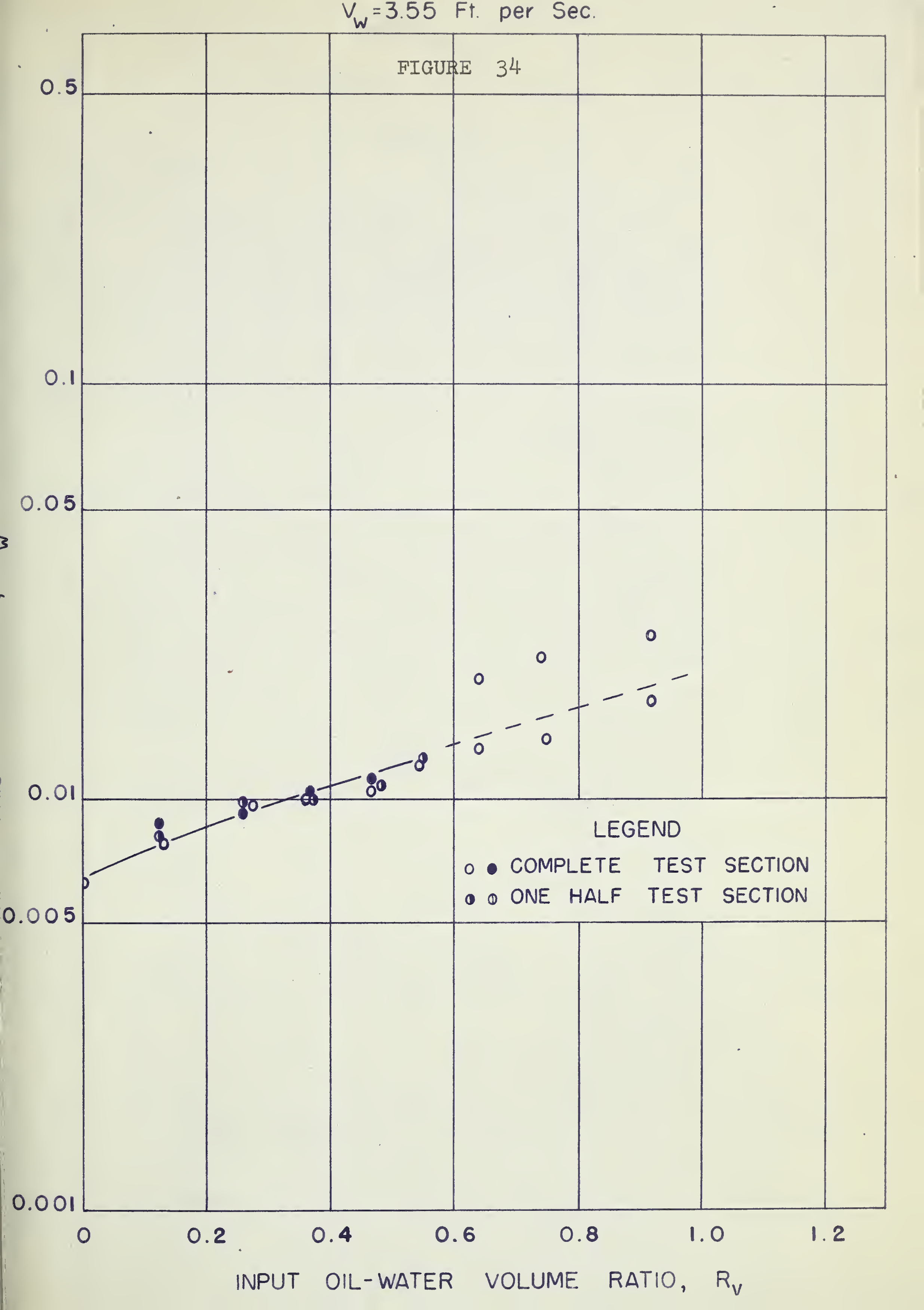


V_w = 1.08 Ft. per Sec.
FIGURE 31
0.5
0.1
0.05
0.01
0.005
0.001
0
0.5
1.0
1.5
2.0
2.5
3.0
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION
INPUT OIL-WATER VOLUME RATIO, R_V

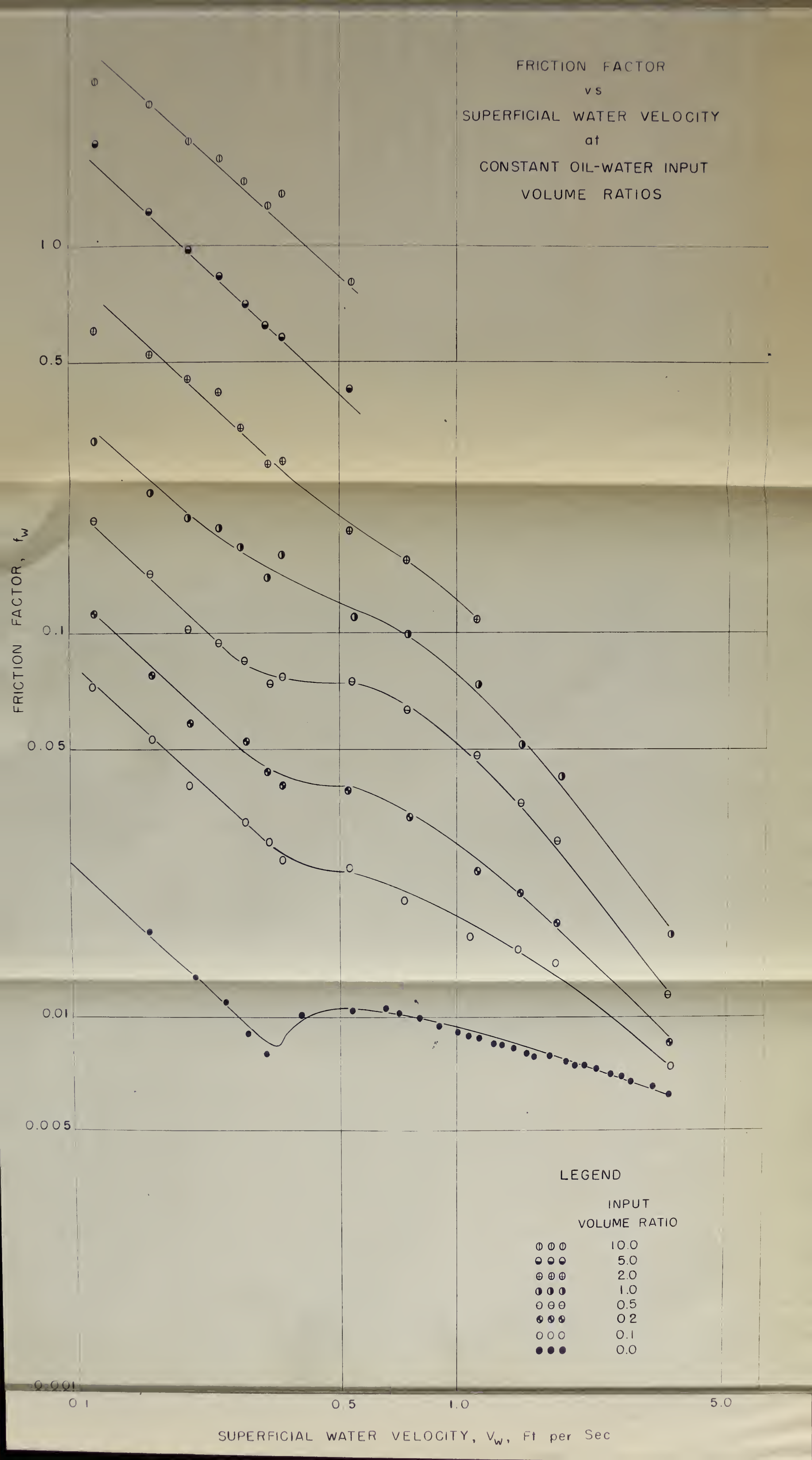


V_w=1.44 Ft. per Sec.
FIGURE 32
0.5
0.1
0.05
0.01
0.005
0.001
FRICTION FACTOR, f_w
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION
0
0.5
1.0
1.5
2.0
2.5
3.0
INPUT OIL-WATER VOLUME RATIO, R_v


V_w = 1.79 Ft. per Sec.
FIGURE 33
0.5
0.1
0.05
0.01
0.005
0.001
0
0.4
0.8
1.2
1.6
2.0
2.4
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION
INPUT OIL-WATER VOLUME RATIO, R_V
FRICTION FACTOR, f_W



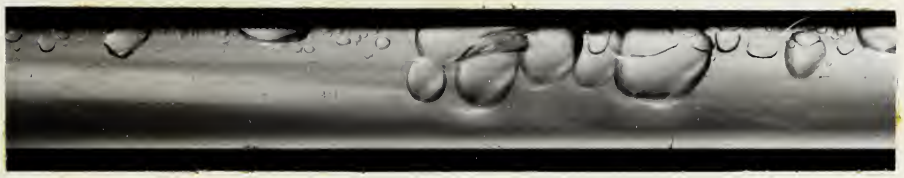
$V_w = 3.55$ Ft. per Sec.
FIGURE 34
0.5
0.1
0.05
0.01
0.005
0.001
LEGEND
COMPLETE TEST SECTION
ONE HALF TEST SECTION
0
0.2
0.4
0.6
0.8
1.0
1.2
INPUT OIL-WATER VOLUME RATIO, R_v


FRICTION FACTOR
vs
SUPERFICIAL WATER VELOCITY
at
CONSTANT OIL-WATER INPUT
VOLUME RATIOS
1.0
0.5
0.1
0.05
0.01
0.005
0.001
FRICTION FACTOR, f_w
0.1
0.5
1.0
5.0
SUPERFICIAL WATER VELOCITY, V_w, Ft per Sec
LEGEND
INPUT
VOLUME RATIO
10.0
5.0
2.0
1.0
0.5
0.2
0.1
0.0

indicates the excellent agreement in general form with the curves developed for the parallel plate flow system. The laminar region family of constant input oil-water volume ratio curves have the same slope, and roughly the same spacing as the theoretical curves shown in Figure 2.

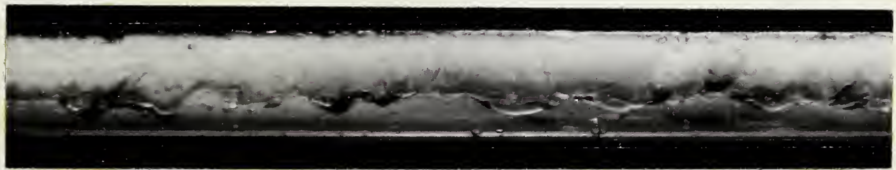
Flow Patterns

The flow patterns were investigated by taking a complete series of photographs for most of the ratios and water rates studied. For the sake of brevity only five sets of photographs are included here but this series is on file. In general three types of flow patterns were observed. At low input oil-water volume ratios a bubble flow was observed. As the input ratio and water rates were increased, bubble flow gradually settled into a laminar two layer flow with a quiet interface. At the higher rates of flow 1.44 ft per sec to 3.55 ft per sec and at higher input ratios the laminar flow changed gradually with increasing agitation at the interface to a completely mixed type of flow. Figure 36 shows the three flow patterns encountered. The qualitative effect of increasing the input oil-water volume ratio is shown photographically in Figures 37, 38, and 39 for three different water rates. A series for increasing superficial water velocities for a constant input ratio is presented in Figure 40. In gas-liquid work, Govier, Radford and Dunn(10) have described flow patterns in conjunction with pressure drop measurements and separated the pressure drop curves into four regimes using maxima and minima points as a basis for division.



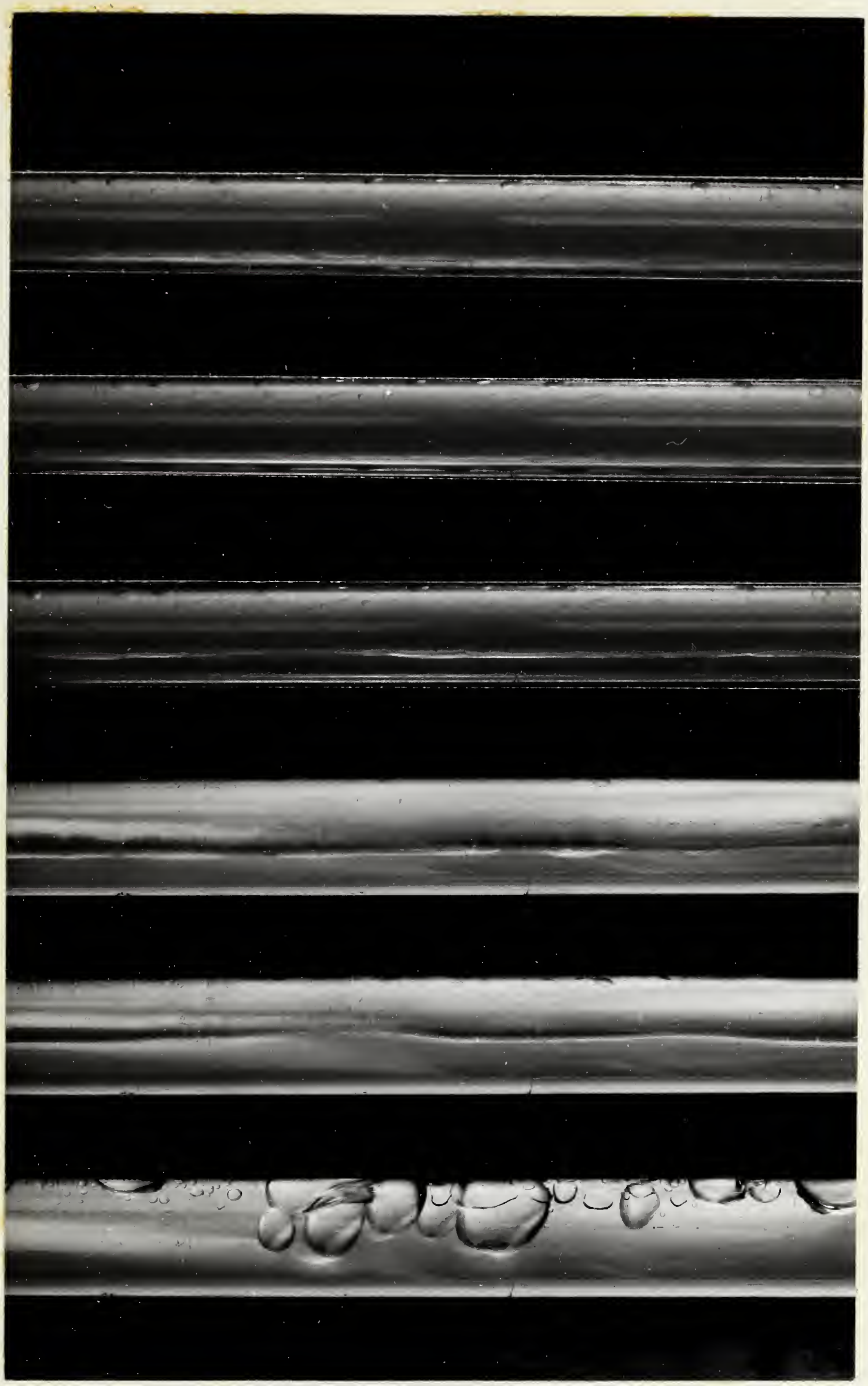
BUBBLE FLOW

LAMINAR FLOW



MIXED FLOW

FIGURE 36



INPUT OIL-WATER VOLUME RATIO

9.10

5.68

1.71

1.01

0.52

0.15

FIGURE 37

VARYING INPUT OIL-WATER VOLUME RATIOS AT

A CONSTANT SUPERFICIAL WATER VELOCITY OF 0.287 FT PER SEC

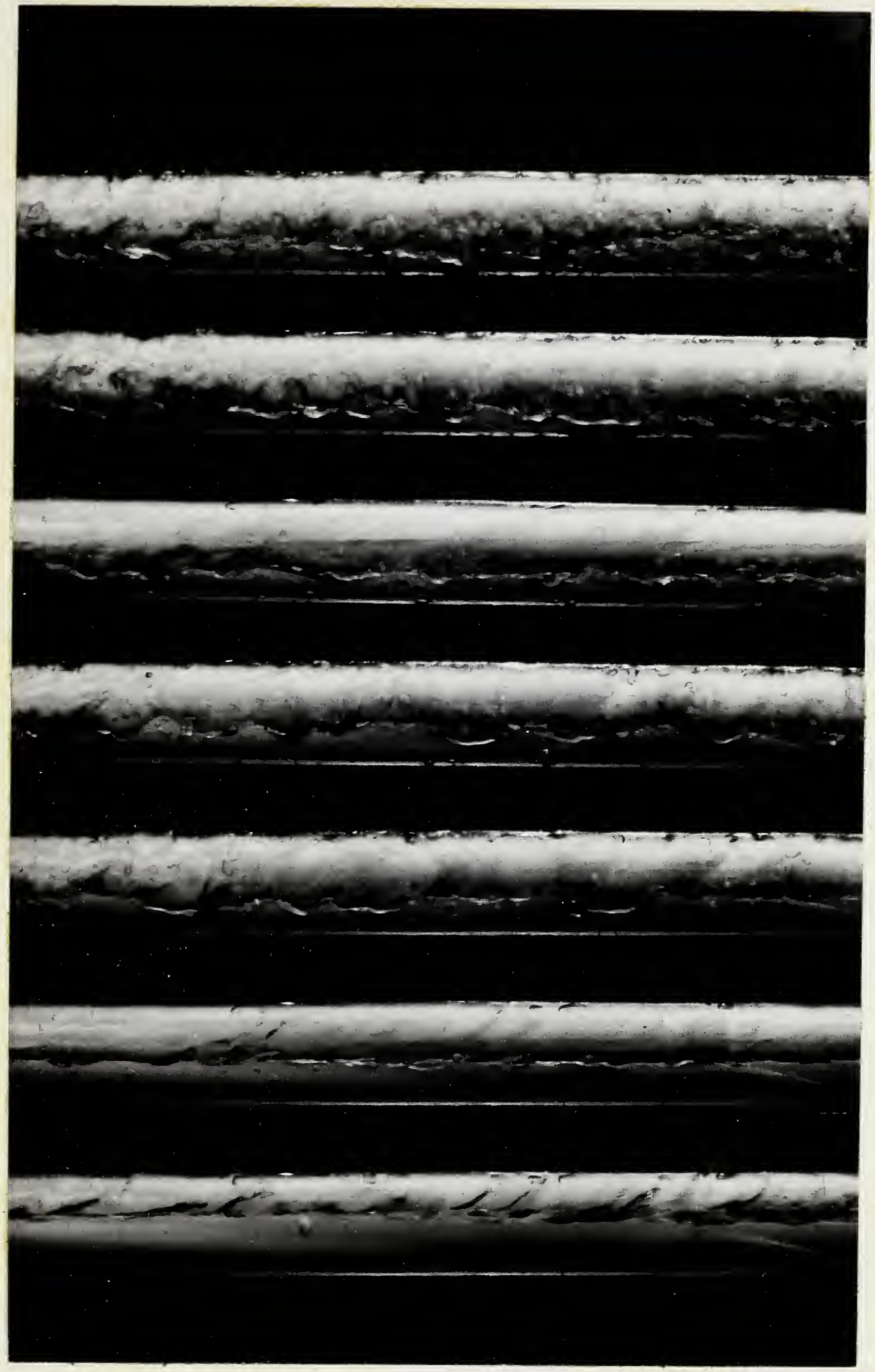
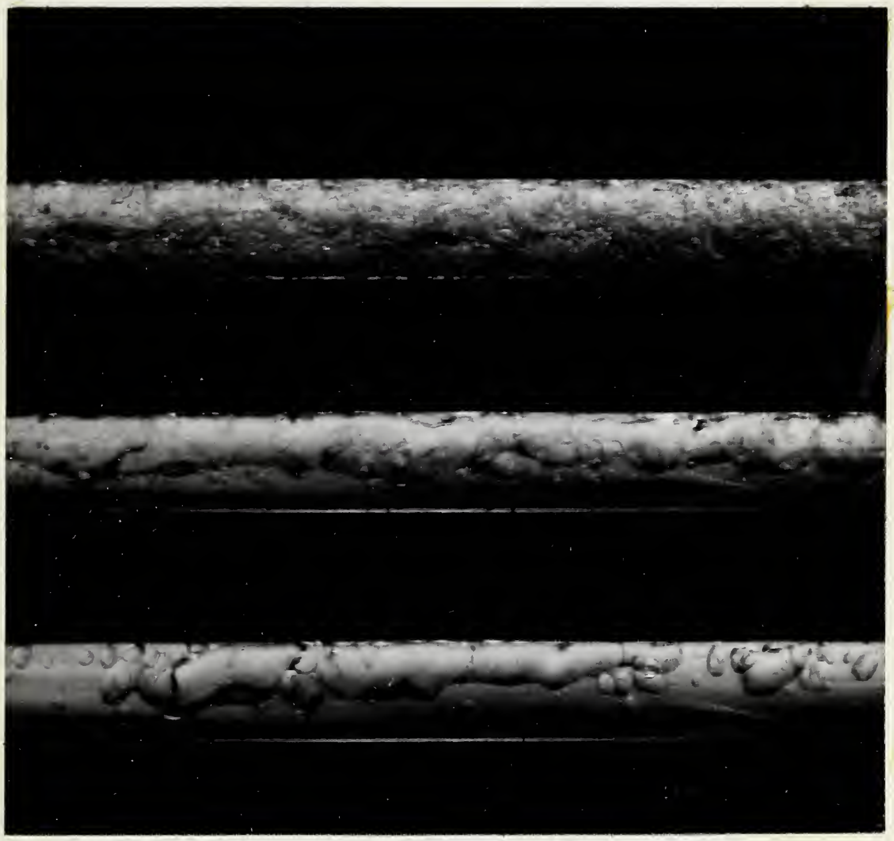


FIGURE 38

VARYING INPUT OIL-WATER VOLUME RATIOS AT

A CONSTANT SUPERFICIAL WATER VELOCITY OF 1.79 FT PER SEC



INPUT OIL-WATER VOLUME RATIO

0.921

0.470

0.137

FIGURE 39

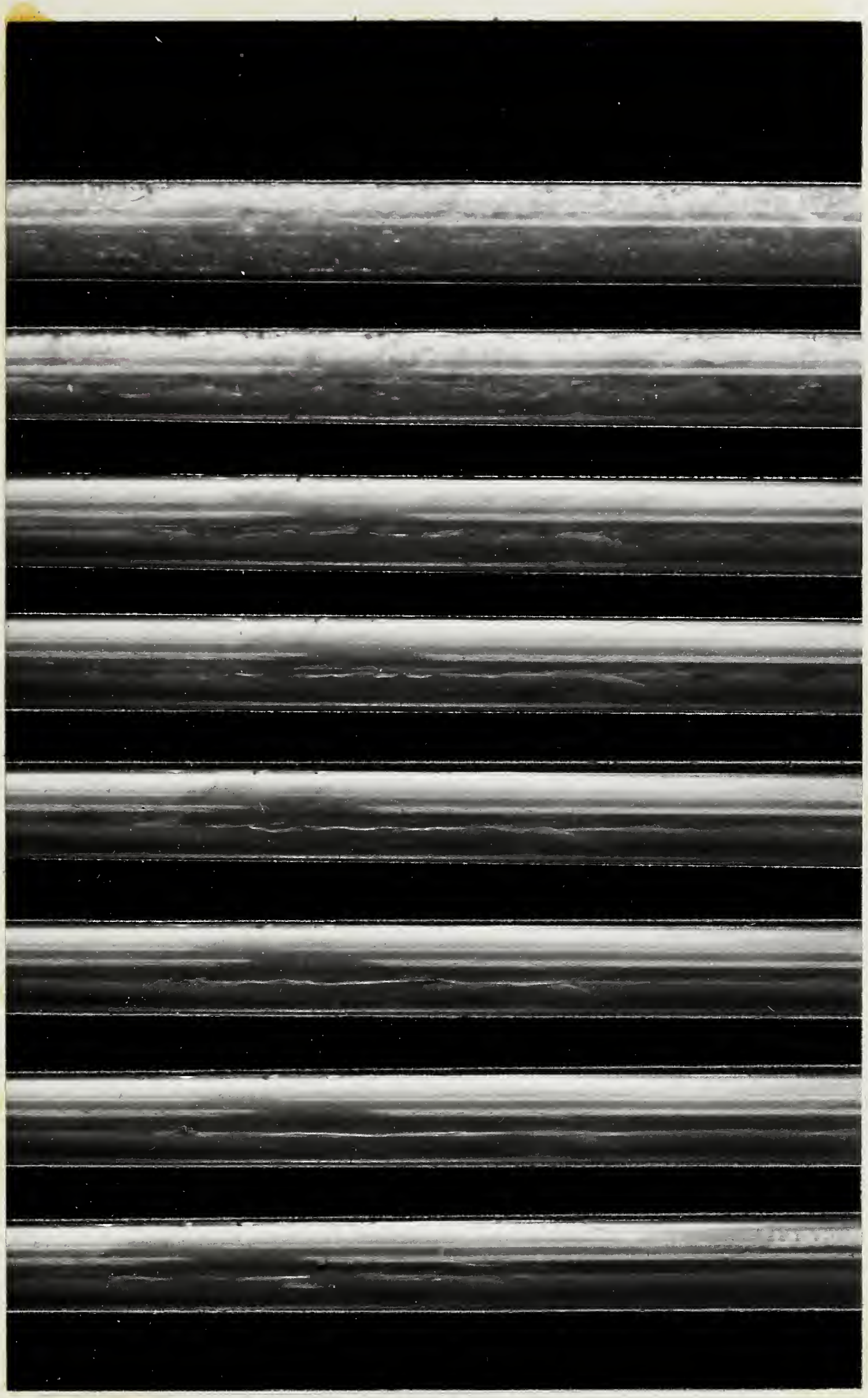
VARYING INPUT OIL-WATER VOLUME RATIOS AT

A CONSTANT SUPERFICIAL WATER VELOCITY OF 3.55 FT PER SEC

FIGURE 16

VARYING SUPERFICIAL WATER VELOCITY AT

A CONSTANT INPUT OIL-WATER VOLUME RATIO OF 1:4



SUPER-
FICIAL
WATER
VELOCITY
FT PER
SEC

3.24

2.52

1.79

1.44

1.08

0.718

0.538

0.358

FIGURE 40

VARYING SUPERFICIAL WATER VELOCITIES AT

A CONSTANT INPUT OIL-WATER VOLUME RATIO OF 1.0

Applying this gas-liquid analogy to the data obtained for the liquid - liquid system it appears from photographic evidence and examination of Figures 9 to 14 which contain the first maxima that bubble flow is occuring. This corresponds to a part of pressure drop Regime I as described by Short (18) the only difference being that the oil is not surrounded by the water but is flowing at the top of the pipe.

The second discontinuity in Figures 19 and 20 appears to be caused by a change from laminar two-layer flow to the completely mixed type of flow. This can be seen from Figure 38 which shows increasing ratios for the superficial water velocity plotted in Figure 20. This type of flow corresponds roughly to that of Regime II described by Short (18) with the exception again that the oil flows on top of the water.

Hold-Up

Hold-up data are plotted as H_R, the hold-up ratio or the ratio of the input oil-water ratio to the in situ oil-water ratio, versus the input oil-water volume ratio. The hold-up data taken in the laminar region for five different water rates (Figure 41) show some scattering but indicate that the hold-up ratio is not a function of velocity but is dependent on the input ratio and viscosity of the two liquids. This is consistent with the theoretical prediction for parallel plates shown in Figure 3.

Data obtained in the turbulent region of flow show in Figure 42 that in this case the superficial water velocity

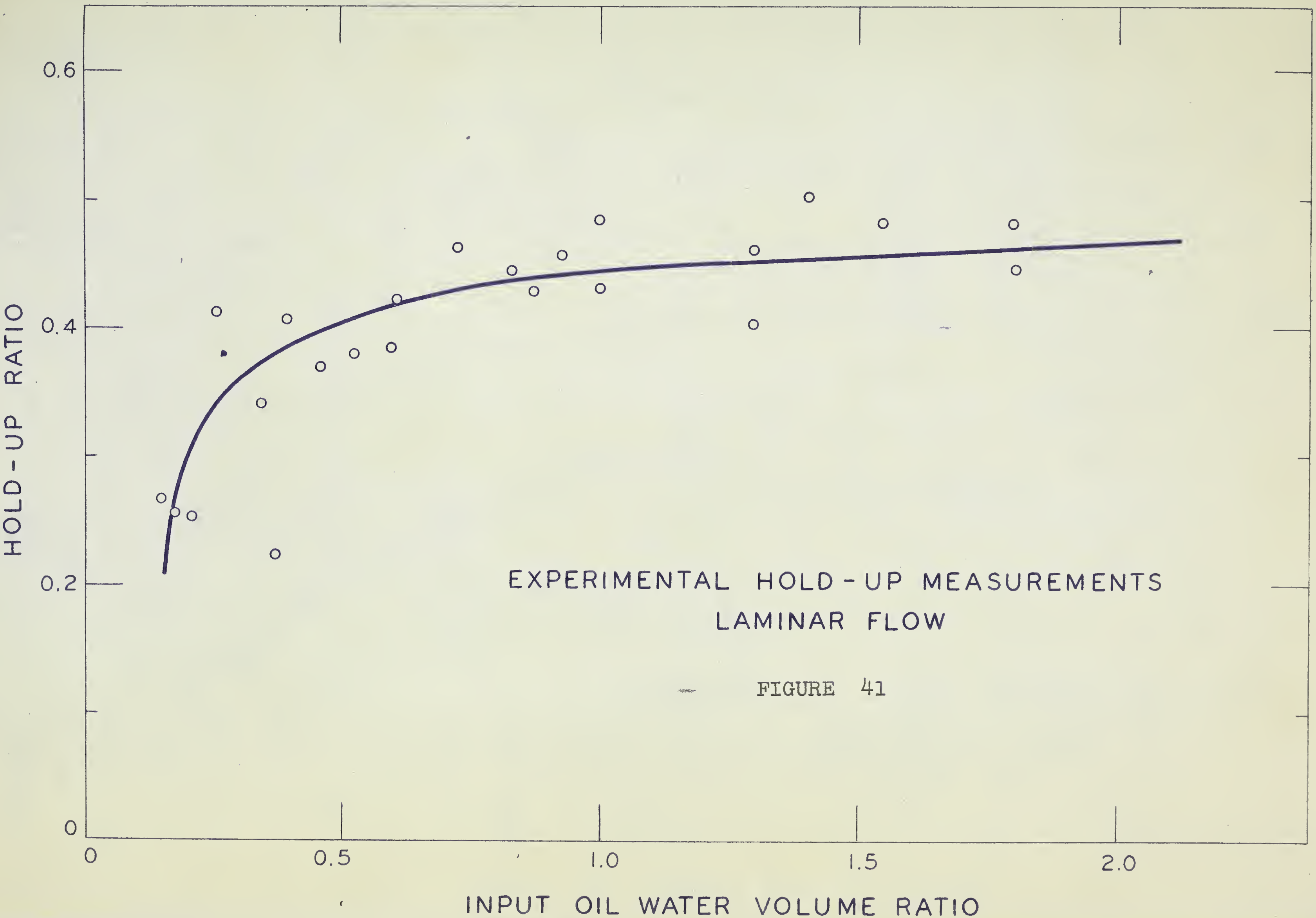
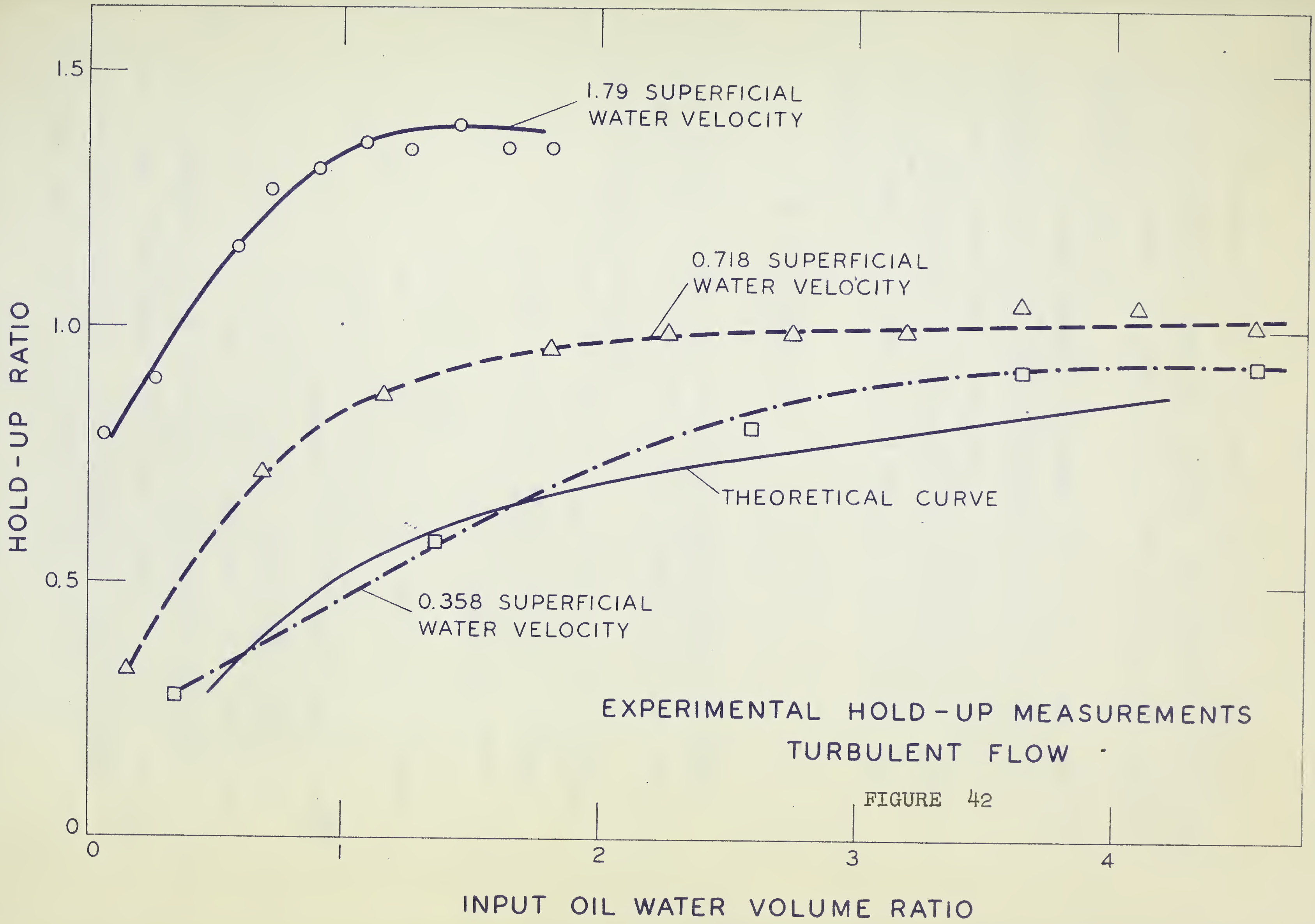

EXPERIMENTAL HOLD - UP MEASUREMENTS
LAMINAR FLOW
HOLD - UP RATIO
0
0.2
0.4
0.6
INPUT OIL WATER VOLUME RATIO
0
0.5
1.0
1.5
2.0

FIGURE 41



EXPERIMENTAL HOLD-UP MEASUREMENTS
TURBULENT FLOW

FIGURE 42

is an important factor. The theoretical curve (Figure 3) is also plotted on this set of axis. The general trend toward zero at an input oil-water volume ratio of zero is shown in Figures 41 and 42.

Govier, Radford and Dunn (10) suggest that for the vertical flow of gas-liquid systems a hold-up of 1.0 is approached at an input ratio of gas to liquid of zero, or in other words when the flowing material is all liquid. They relate hold-up to average liquid and gas velocities by the equation

$$H_R = \frac{V_G - V_L}{V_L} + 1$$

where V_G and V_L are the average gas and liquid velocities. Applying the extreme limits in this equation can lead to two different conclusions:

(i) When the pipe is completely occupied by the liquid, one might expect the slip velocity, $V_G - V_L$, to be zero and the hold-up ratio H_R to be unity

(ii) When the pipe is full of liquid, $V_G = 0$ and the hold-up ratio $H_R = 0$

The data of Govier, Radford and Dunn appear to support unity as the limiting value. On the other hand, the theory developed in the present investigation for parallel plates indicates that the hold-up ratio should approach zero. The experimental results in Figures 41 and 42 strongly support this prediction. It is suggested that the second interpretation (ii) is the correct one.

CONCLUSIONS

(1) Pressure drop data have been correlated by plotting a Fanning type friction factor based on water properties versus a superficial water velocity. The relationship applies only to water and Kremol 70 mineral oil flowing in a one inch horizontal pipe at 77±0.5° F. The correlation shows excellent agreement in general form with a mathematical analysis made of the flow of two immiscible liquids between wide parallel plates.

(2) For the liquid ratios and water rates studied, three flow patterns were observed:

(a) Bubble flow

(b) Laminar flow

(c) Mixed flow

(3) Experimental hold-up measurements are in complete accord with the theory developed for parallel plates. In the laminar region of flow, hold-up is a function of liquid input ratio and the viscosity. In the turbulent region hold-up is also a function of superficial water velocity.

In the limiting case when the input liquid ratio is zero, the hold-up appears to approach zero.

A P P E N D I C E S

APPENDIX A

(EXPERIMENTAL DATA)

A P P E N D I X   A

Table AI shows the pressure drop for pure oil and pure water flowing in the test section. Hold-up measurements and pressure drop data for varying input oil-water volume ratios are tabulated in Table AII for thirteen different superficial water velocities. Pressure drop data marked with an asterisk were obtained by measuring the differential over 14.2 feet of the test section. Hold-up measurements were difficult to make so these data were obtained for only a few selected water velocities.

TABLE A1

PRESSURE DROP DATA

FLOWING TEMPERATURE = 68 $\pm$ 0.5° F

WATER VISCOSITY = 0.894 cp

OIL VISCOSITY = 18.0 cp

TUBE DIAMETER = 0.8057 INCHES

LENGTH OF TEST SECTION = 28.18 FEET

(1)	(2)	(3)	(4)	(5)
FLUID FLOWING	VELOCITY FT PER SEC	PRESSURE DROP INCHES OF WATER		
		FIRST READING	SECOND READING	THIRD READING
water	0.116	0.12	0.12	0.12
	0.162	0.14	0.14	0.14
	0.206	0.17	0.17	0.17
	0.247	0.22	0.22	0.22
	0.287	0.24	0.24	0.24
	0.327	0.28	0.28	0.28
	0.358	0.38	0.42	0.34
	0.538	1.00	0.98	0.98
	0.718	1.56	1.62	1.58
	0.890	2.32	2.30	2.32
	1.08	3.22	3.19	3.17
	1.26	4.14	4.16	4.13
	1.44	5.17	5.17	5.14
	1.62	6.05	6.60	6.45
	1.79	7.85	8.00	7.75
	1.97	9.20	9.30	9.10
	2.15	10.6	10.8	10.5
	2.34	12.5	12.3	12.2
	2.52	13.4	13.9	13.8
	2.69	15.6	15.6	15.5
	2.87	17.5	17.3	17.2
	3.24	20.8	21.5	20.9
	3.55	24.9	24.8	25.0
	4.01	31.6	31.8	31.5
	5.34	51.6	51.6	51.6
	6.78	76.6	75.8	75.7
	8.01	104	103	105
	9.35	135	135	137

(1)	(2)	(3)	(4)	(5)
water	10.7	174	176	174
	12.0	218	214	211
	13.4	267	267	248
oil	0.043	0.608	0.578	0.618
	0.096	1.24	1.20	1.22
	0.150	1.95	1.89	1.91
	0.210	2.65	2.62	2.63
	0.250	3.33	3.36	3.44
	0.724	13.3	13.4	13.3
	0.923	16.6	16.3	16.6
	1.10	19.6	19.6	19.6
	1.30	22.5	22.4	22.7
	1.47	25.5	25.6	25.8
	1.63	28.5	28.6	28.9
	1.79	31.4	31.6	31.8
	1.96	34.4	34.6	35.0
	2.12	37.7	37.7	38.0
	2.28	40.4	41.0	41.1
	2.44	43.5	43.8	44.0
	2.61	46.6	46.8	47.2
	2.94	52.0	52.9	53.0
	3.26	58.1	59.0	58.8
	2.31	42.8	42.8	42.8
	3.80	67.3	69.4	70.0
	5.28	94.5	95.1	95.1
	6.60	119	120	120
	7.92	145	147	146
	9.24	192	193	193
	10.56	290	290	290

TABLE A11

PRESSURE DROP DATA

FLOWING TEMPERATURE = $77 \pm 0.5^{\circ}$ F

WATER VISCOSITY = 0.894 cp

OIL VISCOSITY = 18.0 cp

TUBE DIAMETER = 0.8057 INCHES

LENGTH OF TEST SECTION = 28.18 FEET

(1)	(2)	(3)	(4)	(5)	(6)	(7)
SUPERFICIAL WATER VELOCITY FT PER SEC	INPUT OIL-WATER VOLUME RATIO FT^3 PER FT^3	PRESSURE DROP INCHES OF WATER		IN SITU OIL-WATER VOLUME RATIO FT^3 PER FT^3		HOLD-UP RATIO*
		FIRST READING	SECOND READING	FIRST READING	SECOND READING	
0.116	0.37	0.70	0.75	1.65	1.67	0.224
	0.83	0.90	0.95	1.85	1.88	0.443
	1.29	1.40	1.45	3.21	3.21	0.403
	1.81	2.05	2.00	3.82	4.09	0.446
	2.16	2.80	2.80	3.76	3.96	0.555
	2.50	3.40	3.30	4.69	4.69	0.543
	4.21	6.90	6.90			
	7.96	10.5	11.2			
	11.1	14.7	15.1			
0.162	0.27	1.00	0.90	0.65	0.63	0.413
	0.59	1.05	1.00	1.54	1.53	0.386
	0.93	1.40	1.80	2.02	2.01	0.458
	1.30	2.45	2.45	2.72	2.95	0.461
	1.54	3.20	3.15	3.03	3.23	0.482
	1.79	3.80	3.65	3.69	3.56	0.482
	3.01	7.40	7.30			
	5.70	12.2	11.9			
	8.02	16.2	15.4			
	10.0	20.1	19.6			
0.206	0.21	0.80	0.80	0.83	0.82	0.252
	0.48	1.20	1.20	1.24	1.27	0.370

$$\text{* HOLD-UP RATIO} = \frac{\text{input oil-water volume ratio}}{\text{in situ oil-water volume ratio}}$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.206	0.73	2.05	1.95	1.40	1.47	0.507
	1.02	2.80	2.75	1.80	1.75	0.572
	1.21	3.50	3.40	2.14	1.93	0.634
	1.41	4.25	4.10	2.92	2.69	0.502
	2.37	7.70	7.70			
	4.47	11.9	11.7			
	6.31	16.6	15.4			
	7.91	21.4	19.8			
	9.51	26.3	23.5			
	11.1	30.4	29.0			
0.247	0.17	1.40	1.00	0.69	0.66	0.256
	0.39	1.30	1.20	0.95	0.96	0.408
	0.61	2.20	2.25	1.44	1.43	0.422
	0.85	3.00	2.85	1.66	1.71	0.505
	1.01	3.80	3.60	2.10	2.06	0.485
	1.17	4.40	4.10	2.24	2.24	0.523
	1.97	8.50	8.30			
	3.74	12.7	12.1			
	5.26	16.8	16.0			
	6.60	21.7	20.4			
	7.95	26.1	24.9			
	9.23	30.5	29.5			
0.287	0.15	1.80	1.40	0.56	0.56	0.267
	0.33	1.60	1.60	0.98	0.97	0.341
	0.52	2.40	2.05	1.34	1.37	0.379
	0.73	3.10	3.05	1.55	1.60	0.463
	0.87	3.80	3.65	2.00	2.06	0.429
	1.01	4.60	4.40	2.28	2.30	0.431
	1.71	8.20	8.10			
	3.22	12.1	12.0			
	4.54	16.3	16.3			
	5.68	20.6	20.6			
	6.82	24.7	25.1			
	7.95	30.2	30.1			
	9.10	34.6	34.7			
0.327	0.14	2.20	1.65			
	0.29	2.00	1.70			
	0.46	2.50	2.45			
	0.64	3.40	3.30			
	0.77	4.20	4.05			
	0.89	4.70	4.40			
	1.50	8.30	8.20			
	2.83	12.7	12.9			
	3.98	17.2	17.1			
	5.00	21.8	21.4			
	6.00	25.9	25.7			
	7.00	30.6	30.1			
	8.00	36.3	34.2			

(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.358	1.37	7.2	9.0	2.29	2.32	0.59
	2.58	11.7	13.4	3.19	3.21	0.81
	3.64	16.9	17.5	3.96	3.96	0.92
	4.56	21.1	21.9	4.83	4.92	0.93
	5.48	26.6	26.5	5.67	5.69	0.97
	6.39	31.5	31.4	6.16	6.13	1.04
	7.30	34.9	34.8	7.07	7.33	1.01
	8.22	39.9	40.0	9.32	8.45	0.95
	9.12	45.2	45.1	9.18	9.18	0.99
	1.37	4.5*	4.3*			
	2.58	6.8*	7.0*			
	3.64	8.9*	8.8*			
	4.56	11.2*	11.1*			
	5.48	13.3*	13.1*			
	6.39	15.8*	15.6*			
	7.30	18.1*	18.3*			
	8.22	21.4*	20.2*			
	9.12	23.5*	22.5*			
0.538	0.91	10.5	10.3			
	1.71	15.4	15.2			
	2.42	19.6	19.6			
	3.02	25.1	24.2			
	3.65	30.2	28.6			
	4.24	35.0	33.7			
	4.85	39.0	38.9			
	5.47	43.3	43.6			
	6.06	49.0	49.4			
0.718	0.68	12.9	12.5	0.97	0.91	0.72
	1.28	17.7	17.2	1.48	1.47	0.87
	1.81	22.9	21.7	1.87	1.95	0.95
	2.26	28.5	28.1	2.28	2.28	1.00
	2.73	36.3	33.5	2.74	2.81	0.98
	3.17	41.4	38.9	3.15	3.38	1.05
	4.08	49.6	49.6	3.95	3.80	1.05
	4.54	54.4	55.0	4.48	4.43	1.02
	0.68	7.0*	7.1*			
	1.28	9.5*	9.7*			
	1.81	11.8*	11.7*			
	2.26	14.5*	13.5*			
	2.73	17.1*	16.2*			
	3.17	19.6*	18.5*			
	3.63	22.0*	20.6*			
	4.08	25.2*	23.7*			
	4.54	27.6*	26.2*			

* Over one half test section.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.08	0.45	19.7	14.1			
	0.85	24.1	19.1			
	1.20	30.1	27.3			
	1.51	35.4	31.7			
	1.82	38.9	37.8			
	2.11	43.7	43.0			
	2.42	48.2	46.2			
	2.72	53.4	52.0			
	3.03	48.5	54.0			
	0.45	9.2*	8.0*			
	0.85	11.2*	12.1*			
	1.20	13.6*	14.9*			
	1.51	15.3*	17.7*			
	1.81	17.5*	21.4*			
	2.11	20.3*	25.1*			
	2.42	22.7*	26.2*			
	2.72	24.9*	24.8*			
	3.03	28.1*	30.1*			
1.44	0.34	19.0	19.3			
	0.64	23.9	23.8			
	0.90	29.1	29.8			
	1.13	34.6	36.7			
	1.36	38.5	41.6			
	1.58	39.1	39.7			
	1.81	33.1	38.2			
	2.04	29.8	42.4			
	2.26	31.8	32.0			
	0.34	11.2*	9.5*			
	0.64	13.4*	12.6*			
	0.90	15.3*	15.7*			
	1.13	18.7*	18.4*			
	1.36	20.9*	21.4*			
	1.58	21.8*	24.5*			
	1.81	23.2*	27.0*			
	2.04	21.5*	29.7*			
	2.26	22.3*	29.9*			
1.79	0.27	20.2	25.9	0.29	0.31	0.90
	0.51	26.3	31.6	0.45	0.43	1.16
	0.73	32.5	39.1	0.58	0.56	1.27
	0.91	38.7	35.8	0.69	0.70	1.31
	1.09	31.3	30.1	0.78	0.81	1.37
	1.27	35.8	34.1	0.96	0.93	1.34
	1.45	41.0	33.4	1.03	1.04	1.39
	1.64	39.5	37.5	1.22	1.21	1.35
	1.82	37.5	41.5	1.31	1.38	1.39

* Over one half test section.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
1.79	0.27	13.0*	11.6*			
	0.51	14.7*	14.7*			
	0.73	18.3*	18.0*			
	0.91	21.6*	21.2*			
	1.09	24.6*	24.3*			
	1.27	24.5*	26.7*			
	1.45	22.5*	24.0*			
	1.64	18.7*	20.0*			
	1.82	21.0*	19.8*			
3.55	0.14	31.7	35.3			
	0.26	35.2	40.1			
	0.37	39.3	42.3			
	0.47	44.2	45.7			
	0.55	48.5				
	0.64	53.2				
	0.74	57.2				
	0.83	61.7				
	0.92	68.1				
	0.14	16.4*				
	0.26	19.7*				
	0.37	20.3*				
	0.47	22.9*				
	0.55	24.8*				
	0.64	40.4*				
	0.74	46.5*				
	0.83	52.5*				

* Over one half test section.

APPENDIX B

(CALCULATED DATA)

APPENDIX B

Reynolds number and the conventional friction factor have been calculated for experimental runs on the pure liquids and are tabulated in Table Bl. Friction factors calculated by using water properties and superficial water velocities are tabulated for the thirteen water rates in Table Bll. Results marked with an asterisk indicate measurements made over one half the test section.

APPENDIX B

SAMPLE CALCULATIONS

(1) Measurements of Test Section

$$\text{Volume (1)} = 3786\ \text{cm}^3$$

$$\text{Volume (2)} = 3785\ \text{cm}^3$$

$$\text{Total Length} = 37.71\ \text{feet}$$

$$D^2 = \frac{3785\ (4)}{16.4\ (\pi)\ 1728\ (37.71)}$$

$$D = 0.0671\ \text{feet}$$

$$\text{Area} = \frac{\pi D^2}{4} = 0.003538\ \text{ft}^2$$

$$\text{Length of Test Section} = 28.18\ \text{feet}$$

(2) Conversion of lbs per min to ft per sec

(a) Water

$$1\ \frac{\text{lb}}{\text{min}} = 0.0756\ \frac{\text{ft}}{\text{sec}}$$

(b) Oil

$$1\ \frac{\text{lb}}{\text{min}} = 0.0905\ \frac{\text{ft}}{\text{sec}}$$

(3) Calculation of Reynolds Number

$$Re = \frac{DV\rho}{\mu}$$

(a) Water @ 20° C

$$Re = \frac{0.8057\ (62.4)\ \ V\ \ (10)^4}{12\ \ (6.72)\ \ (1.00)}$$

$$= 6.24\ \ (10)^3\ \ V$$

(b) Water @ 25° C

$$Re = 6.97\,(10)^3\,V$$

(c) Kremol 70 @ 20° C

$$Re = \frac{0.8057\,(62.4)\,V\,(10)^4}{12\,(22.5)\quad 6.72}$$

$$Re = 232\,V$$

(d) Kremol 70 @ 25° C

$$Re = 290\,V$$

(4) Calculation of Friction Factor

$$\Delta P' = \frac{2\,f\Delta L\,V^2}{\rho\,g\,c\,D} \quad ; \quad P' \text{ in inches of water}$$

(a) Water

$$f = \frac{\Delta P'\,(32.14)\,0.8057}{12\,(2)\,28.16\,(12)\,V^2}$$

$$f = \frac{\Delta P'}{312\,V^2}$$

(b) Oil

$$f = \frac{\Delta P'\,(32.14)\,0.8057}{12\,(2)\,28.16\,(12)\,0.836}$$

$$f = \frac{\Delta P'}{261\,V^2}$$

TABLE B1

CALCULATED FRICTION FACTOR AND REYNOLDS NUMBER

WATER VISCOSITY = 0.894 cp @ 25° C

OIL VISCOSITY = 18.0 cp @ 25° C

TUBE DIAMETER = 0.8057 INCHES

LENGTH OF TEST SECTION = 28.18 INCHES

(1) FLUID FLOWING	(2) VELOCITY FT PER SEC	(3) FRICTION FACTOR FIRST PRESSURE DROP	(4) FRICTION FACTOR SECOND PRESSURE DROP	(5) FRICTION FACTOR THIRD PRESSURE DROP	(6) REYNOLDS NUMBER $\frac{DV\rho}{\mu}$
WATER	0.116	0.028	0.028	0.028	724
	0.162	0.017	0.017	0.017	1011
	0.206	0.012	0.012	0.012	1285
	0.247	0.011	0.011	0.011	1541
	0.287	0.0092	0.0092	0.0092	1791
	0.327	0.0082	0.0082	0.0082	2040
	0.358	0.0097	0.0108	0.0093	2230
	0.538	0.0094	0.0104	0.0104	3350
	0.718	0.0097	0.0101	0.0098	4480
	0.890	0.0091	0.0090	0.0091	5550
	1.08	0.0089	0.0088	0.0088	6730
	1.26	0.0082	0.0083	0.0082	7860
	1.44	0.0080	0.0080	0.0080	8980
	1.62	0.0074	0.0080	0.0079	10100
	1.79	0.0079	0.0081	0.0078	11100
	1.97	0.0076	0.0077	0.0075	12300
	2.15	0.0073	0.0074	0.0073	13400
	2.34	0.0074	0.0072	0.0072	14600
	2.52	0.0069	0.0071	0.0070	15700
	2.69	0.0069	0.0069	0.0068	16800
	2.87	0.0068	0.0067	0.0066	17900
	3.24	0.0064	0.0066	0.0064	20200
	3.55	0.0061	0.0061	0.0062	22100
	4.01	0.0063	0.0063	0.0063	25000
	5.34	0.0057	0.0057	0.0057	33300
	6.78	0.0055	0.0054	0.0054	42300
	8.01	0.0051	0.0051	0.0052	49900
	9.35	0.0049	0.0049	0.0050	58300
	10.7	0.0049	0.0049	0.0049	66700
	12.0	0.0049	0.0048	0.0047	74800
	13.4	0.0047	0.0047	0.0045	83600

(1)	(2)	(3)	(4)	(5)	(6)
OIL	0.043	1.23	1.16	1.25	13.1
	0.096	0.517	0.500	0.508	29.1
	0.150	0.321	0.322	0.326	45.6
	0.210	0.230	0.228	0.229	63.8
	0.250	0.204	0.206	0.211	76.0
	0.724	0.098	0.098	0.098	167
	0.923	0.075	0.073	0.075	214
	1.10	0.057	0.057	0.057	257
	1.30	0.051	0.051	0.052	302
	1.47	0.045	0.046	0.046	340
	1.63	0.041	0.041	0.042	378
	1.79	0.038	0.038	0.038	418
	1.96	0.035	0.036	0.035	453
	2.12	0.032	0.032	0.032	491
	2.28	0.030	0.030	0.030	529
	2.44	0.028	0.028	0.028	567
	2.61	0.026	0.026	0.027	605
	2.94	0.023	0.024	0.024	680
	3.27	0.021	0.021	0.021	756
	2.31	0.031	0.031	0.031	536
	3.80	0.018	0.018	0.018	881
	5.28	0.012	0.013	0.013	1230
	6.60	0.010	0.010	0.010	1530
	7.92	0.0088	0.0088	0.0088	1840
	9.24	0.0086	0.0086	0.0086	2150
	10.56	0.0100	0.0100	0.0100	2450

TABLE B11

CALCULATED FRICTION FACTOR

WATER VISCOSITY = 0.894 cp @ 25° C

OIL VISCOSITY = 18.0 cp @ 25° C

TUBE DIAMETER = 0.8057 INCHES

LENGTH OF TEST SECTION = 28.18 INCHES

(1)	(2)	(3)	(4)
SUPERFICIAL WATER VELOCITY	INPUT OIL-WATER VOLUME RATIO	WATER FRICTION FACTOR, f_W	
FT PER SEC	FT^3 PER FT^3	FIRST PRESSURE DROP	SECOND PRESSURE DROP
0.116	0.37	0.167	0.178
	0.83	0.217	0.226
	1.29	0.333	0.345
	1.81	0.476	0.488
	2.16	0.667	0.667
	2.50	0.810	0.786
	4.21	1.64	1.64
	7.96	2.50	2.65
	11.10	3.50	3.59
0.162	0.27	0.123	0.111
	0.59	0.129	0.123
	0.93	0.173	0.222
	1.30	0.302	0.302
	1.54	0.394	0.388
	1.79	0.468	0.450
	3.01	0.912	0.900
	5.70	1.50	1.47
	8.02	2.00	1.90
	10.00	2.48	2.42
0.206	0.21	0.061	0.061
	0.47	0.091	0.091
	0.73	0.156	0.149
	1.02	0.214	0.210
	1.21	0.267	0.259
	1.41	0.324	0.313
	2.37	0.586	0.586
	4.47	0.905	0.890

(1)	(2)	(3)	(4)
0.206	6.31	1.26	1.17
	7.91	1.64	1.51
	9.51	2.00	1.80
	11.10	2.31	2.21
0.247	0.174	0.072	0.052
	0.389	0.068	0.063
	0.607	0.115	0.118
	0.850	0.158	0.150
	1.012	0.200	0.189
	1.18	0.231	0.215
	1.97	0.446	0.435
	3.74	0.668	0.636
	5.26	0.881	0.842
	6.60	1.14	1.08
	7.95	1.37	1.31
	9.23	1.60	1.55
0.287	0.150	0.070	0.055
	0.334	0.062	0.062
	0.523	0.094	0.080
	0.732	0.121	0.119
	0.871	0.148	0.143
	1.01	0.180	0.172
	1.71	0.321	0.320
	3.22	0.473	0.470
	4.54	0.640	0.640
	5.68	0.808	0.808
	6.82	0.970	0.982
	7.95	1.18	1.18
	9.10	1.36	1.36
0.327	0.135	0.066	0.049
	0.294	0.060	0.051
	0.458	0.075	0.073
	0.643	0.102	0.099
	0.765	0.126	0.121
	0.887	0.141	0.132
	1.50	0.248	0.246
	2.83	0.380	0.387
	3.98	0.515	0.512
	5.00	0.654	0.642
	6.00	0.776	0.768
	7.00	0.916	0.918
	8.00	1.09	1.03
0.358	1.37	0.185	0.231
	2.58	0.301	0.345
	3.64	0.436	0.450
	4.56	0.542	0.564
	5.48	0.682	0.681
	6.39	0.810	0.810

(1)	(2)	(3)	(4)
0.358	7.30	0.895	0.895
	8.22	1.03	1.03
	9.12	1.16	1.16
	1.37	0.231*	0.221*
	2.58	0.349*	0.360*
	3.64	0.456*	0.452*
	4.56	0.570*	0.570*
	5.48	0.672*	0.670*
	6.39	0.812*	0.800*
	7.30	0.928*	0.940*
	8.22	1.10*	1.04*
	9.12	1.21*	1.15*
0.538	0.91	0.112	0.100
	1.71	0.165	0.163
	2.42	0.210	0.210
	3.02	0.269	0.259
	3.65	0.322	0.306
	4.24	0.374	0.360
	4.85	0.417	0.417
	5.47	0.462	0.466
	6.06	0.524	0.528
0.718	0.68	0.080	0.078
	1.28	0.110	0.107
	1.81	0.142	0.135
	2.26	0.177	0.175
	2.73	0.226	0.208
	3.17	0.256	0.242
	3.63	0.267	0.267
	4.08	0.308	0.308
	4.54	0.336	0.342
	0.68	0.087*	0.088*
	1.28	0.117*	0.120*
	1.81	0.146*	0.146*
	2.26	0.180*	0.168*
	2.73	0.212*	0.201*
	3.17	0.243*	0.229*
	3.63	0.273*	0.258*
	4.08	0.312*	0.296*
	4.54	0.345*	0.325*
1.08	0.45	0.054	0.039
	0.85	0.066	0.054
	1.20	0.083	0.076

* Friction factor calculated with data from one half test section.

(1)	(2)	(3)	(4)
1.08	1.51	0.098	0.087
	1.82	0.107	0.104
	2.11	0.121	0.119
	2.42	0.113	0.127
	2.72	0.147	0.144
	3.03	0.134	0.149
	0.45	0.051*	0.044*
	0.85	0.062*	0.067*
	1.20	0.075*	0.082*
	1.51	0.084*	0.098*
	1.82	0.097*	0.117*
	2.11	0.112*	0.140*
	2.42	0.125*	0.145*
	2.72	0.138*	0.157*
	3.03	0.156*	0.164*
1.44	0.34	0.030	0.030
	0.64	0.037	0.037
	0.90	0.045	0.045
	1.13	0.057	0.054
	1.36	0.064	0.060
	1.58	0.061	0.061
	1.81	0.059	0.052
	2.04	0.066	0.045
	2.26	0.050	0.049
	0.34	0.035*	0.030*
	0.64	0.042*	0.039*
	0.90	0.048*	0.049*
	1.13	0.059*	0.057*
	1.36	0.062*	0.064*
	1.58	0.068*	0.076*
	1.81	0.073*	0.084*
	2.04	0.067*	0.092*
	2.26	0.070*	0.093*
1.79	0.27	0.020	0.026
	0.51	0.026	0.031
	0.73	0.032	0.039
	0.91	0.038	0.035
	1.09	0.031	0.030
	1.27	0.035	0.034
	1.45	0.040	0.033
	1.64	0.039	0.037
	1.82	0.037	0.041
	0.27	0.026*	0.023*
	0.51	0.029*	0.029*

* Friction factor calculate d with data from one half test section.

(1)	(2)	(3)	(4)
1.79	0.73	0.036*	0.036*
	0.91	0.043*	0.043*
	1.09	0.049*	0.049*
	1.27	0.049*	0.053*
	1.45	0.044*	0.048*
	1.64	0.037*	0.039*
	1.82	0.042*	0.042*
3.55	0.137	0.0078	0.0087
	0.260	0.0095	0.0092
	0.368	0.0097	0.010
	0.470	0.0105	0.010
	0.553	0.012	
	0.642	0.013	
	0.740	0.014	
	0.828	0.015	
	0.921	0.017	
	0.137	0.0080*	
	0.260	0.0096*	
	0.368	0.0099*	
	0.470	0.0112*	
	0.553	0.0197*	
	0.642	0.0223*	
	0.740	0.0251*	

* Friction factor calculated with data from one half test section.

APPENDIX C

(CALIBRATIONS)

APPENDIX C

The Fisher Porter rotameters were calibrated by time weight measurements and the results are shown in Figures 43 and 44. The three curves on each graph represent:

(1) The calibration of the large rotameter with a steel float

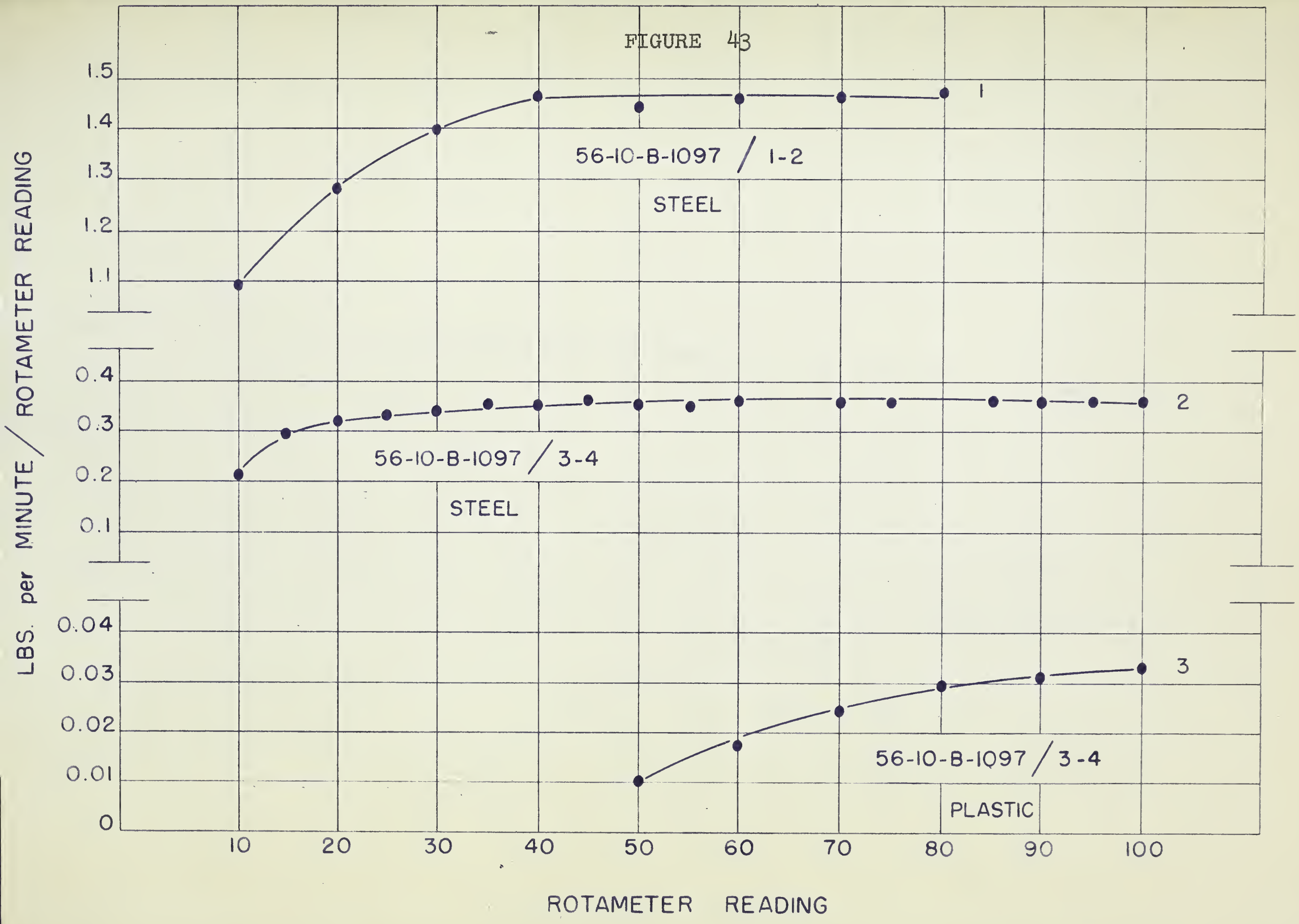
(2) The calibration of the small rotameter with a steel float

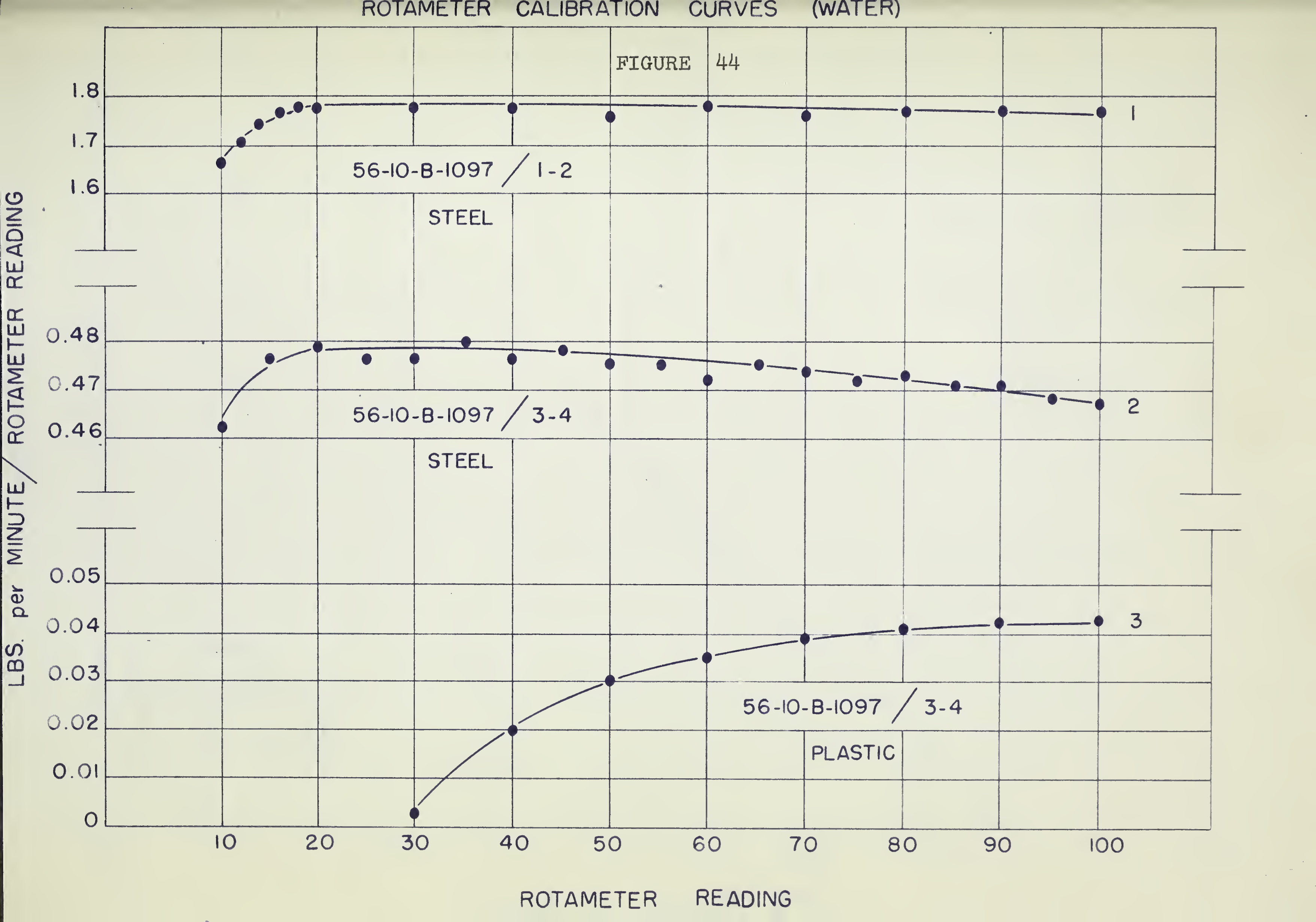
(3) The calibration of the small rotameter with a plastic float

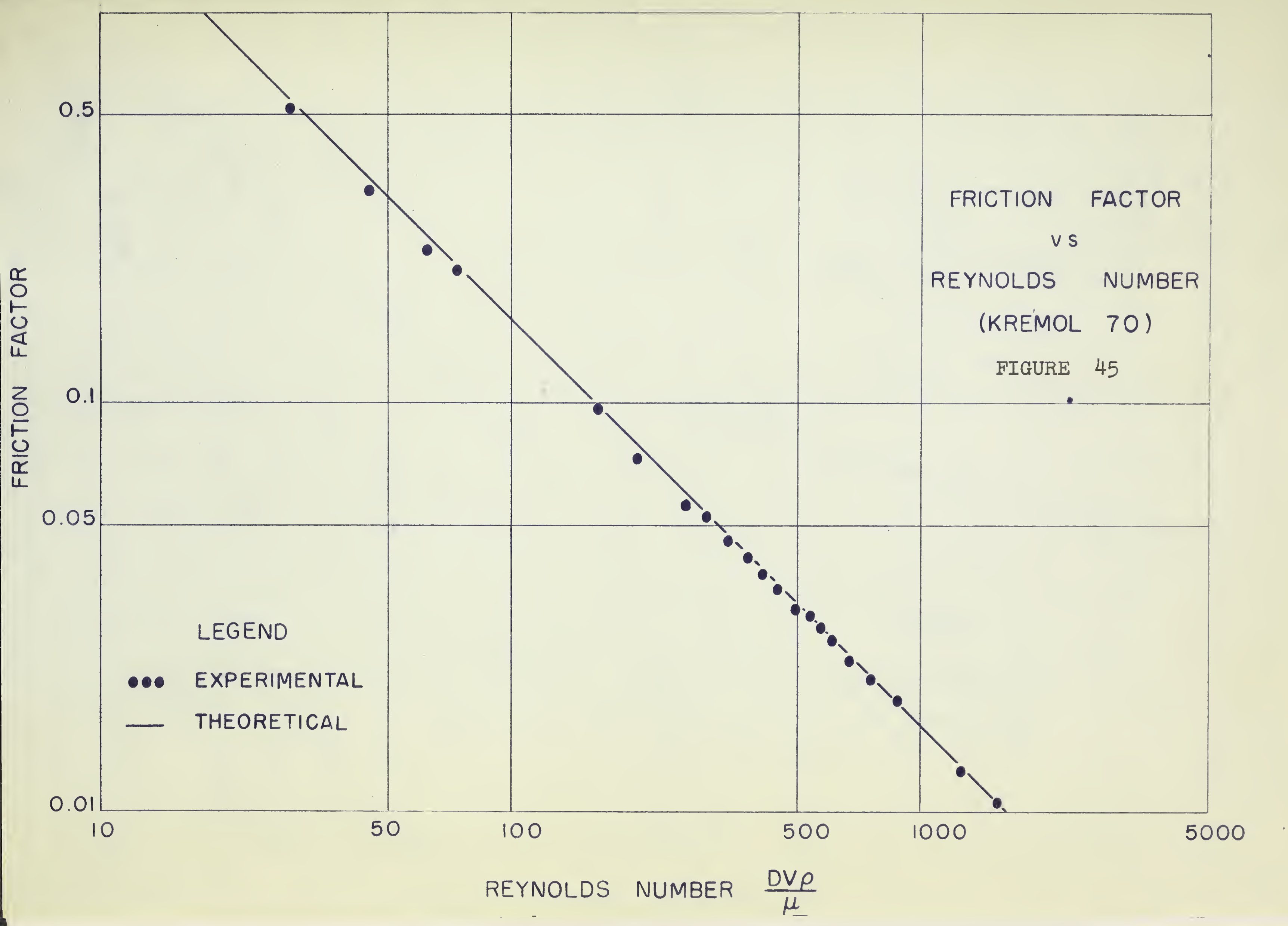
The pipe test section was calibrated by making pressure drop measurements for the oil and water flowing alone. This data is plotted in Figures 45 and 46 on the conventional friction factor versus Reynolds number coordinates.

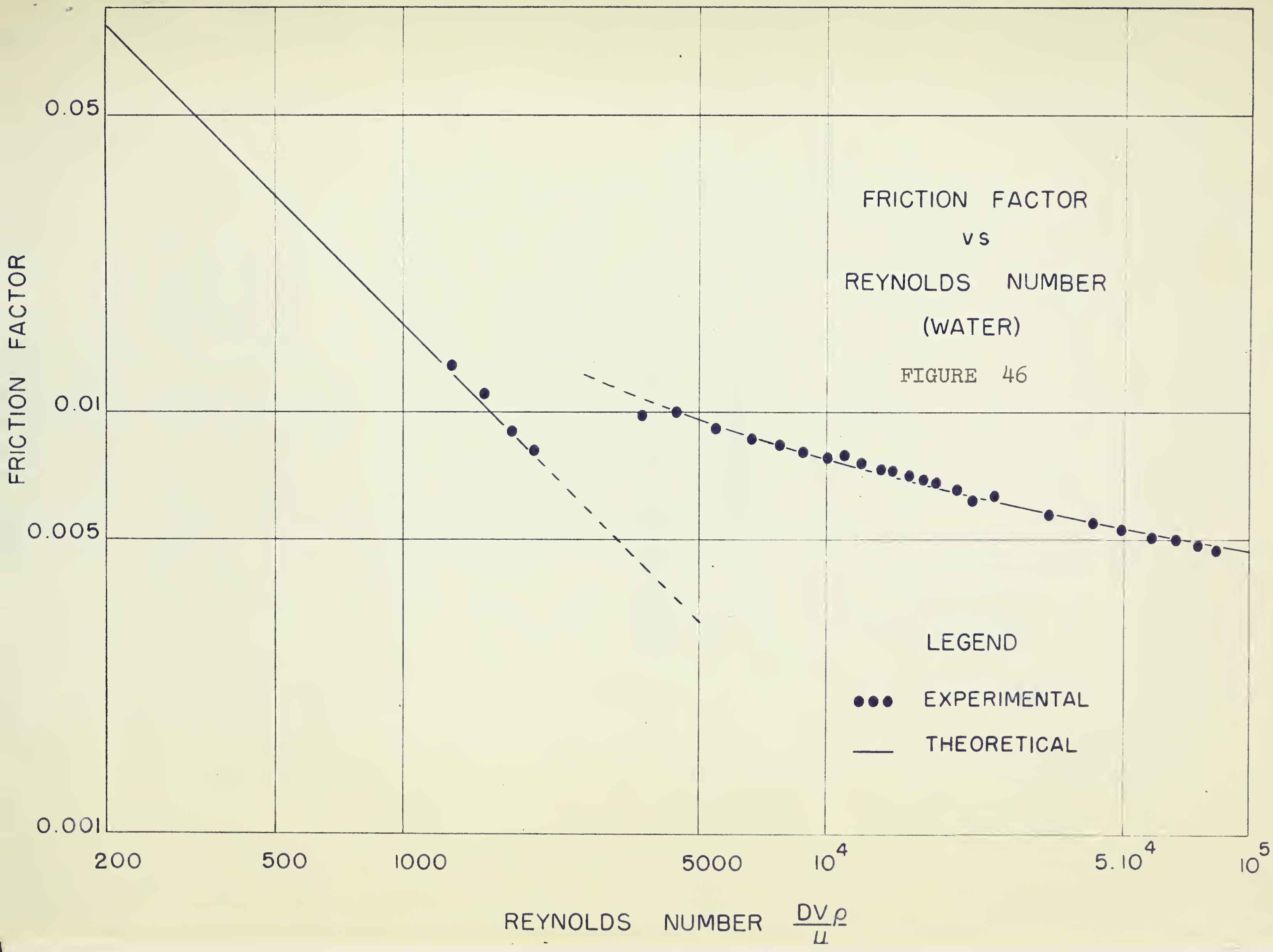
ROTAMETER CALIBRATION CURVES (OIL)

FIGURE 43




ROTAMETER CALIBRATION CURVES (WATER)
FIGURE 44
56-10-B-1097 / 1-2
STEEL
56-10-B-1097 / 3-4
STEEL
56-10-B-1097 / 3-4
PLASTIC
1
2
3
1.8
1.7
1.6
0.48
0.47
0.46
0.05
0.04
0.03
0.02
0.01
0
10
20
30
40
50
60
70
80
90
100
ROTAMETER READING
LBS. per MINUTE / ROTAMETER READING


FRICTION FACTOR
VS
REYNOLDS NUMBER
(KREMOL 70)
FIGURE 45
LEGEND
EXPERIMENTAL
THEORETICAL
FRICTION FACTOR
0.5
0.1
0.05
0.01
10
50
100
500
1000
5000
REYNOLDS NUMBER $\frac{DV\rho}{\mu}$



FRICTION FACTOR
vs
REYNOLDS NUMBER
(WATER)
FIGURE 46
LEGEND
EXPERIMENTAL
THEORETICAL
FRICTION FACTOR
0.05
0.01
0.005
0.001
200
500
1000
5000
10^4
5.10^4
10^5
REYNOLDS NUMBER DVρ/μ

APPENDIX D

(THEORETICAL ANALYSIS)

A P P E N D I X   D

The complete development relating pressure drops of two immiscible liquids to the flow rate, viscosities and geometry of the system is presented in Part 1 for laminar liquid-liquid flow between wide parallel plates. The equations have been integrated for in situ oil-water volume ratios from 0.1 to 10.0 and the results are presented in Table D 1 and D 11.

A sample calculation evaluating Q_A and Q_B for an in situ liquid A-liquid B volume ratio of 2.0 is shown in Part 11. A calculation of the input liquid A-liquid B volume ratio for this insitu ratio is also shown.

For water and Kremol 70 a sample calculation of input oil-water volume ratios, R_V, is shown in Part 111 for an in situ oil-water volume ratio of 2.0. The input oil-water volume ratio was calculated for seven in situ oil-water volume ratios from 0.1 to 10.0 and the results are tabulated in Table D 111.

Part 1V shows the calculation of the parallel plate friction factor defined in the theory by Equation 10. Values for K were calculated for three different water rates and seven in situ oil-water volume ratios and are tabulated in Table D 1V.

PART 1

LAMINAR LIQUID-LIQUID FLOW BETWEEN WIDE PARALLEL PLATES

DIAGRAM

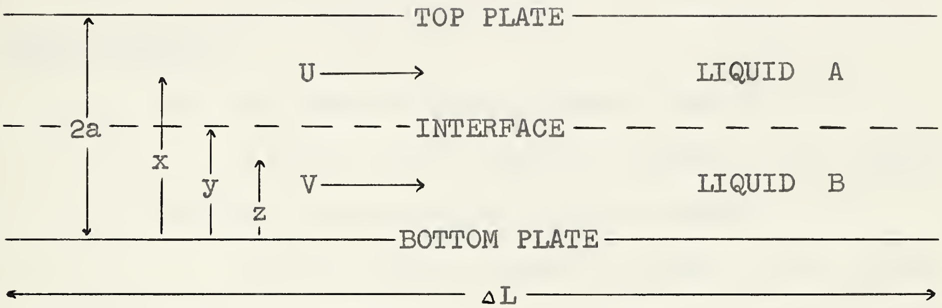


FIGURE 1

NOMENCLATURE

$2a$	=	distance between plates, ft
b	=	width, ft
ΔL	=	length, ft
y	=	distance from the bottom plate to the interface
x	=	distance from the bottom plate to a point in liquid A
z	=	distance from the bottom plate to a point in liquid B
U	=	velocity of liquid A, ft per sec
V	=	velocity of liquid B, ft per sec
Q_A	=	volumetric flow rate of liquid A, ft^3 per sec
Q_B	=	volumetric flow rate of liquid B, ft^3 per sec
μ_A	=	viscosity of liquid A, lbs_M per ft-sec
μ_B	=	viscosity of liquid B, lbs_M per ft-sec
ΔP	=	total pressure drop over the length ΔL, lbs_F per ft^2
R_x	=	unit shear force in the liquid at distance x, lbs_F per ft^2

R_z = unit shear force in the liquid at distance z, lbs_F per ft^2

R_i = unit shear force at the interface of the two liquids, lbs_F per ft^2

g_c = dimensional conversion factor, lbs_M ft per lbs_F sec^2

FORCE BALANCE

(i) on layer between planes x and y

$$\Delta P\,(x - y)\,b - R_x \Delta Lb + R_i \Delta Lb = 0 \quad \ldots D(1)$$

(ii) on layer between planes z and y

$$\Delta P\,(y - z)\,b - R_z \Delta Lb - R_i \Delta Lb = 0 \quad \ldots D(2)$$

The force balance assumes that the bottom layer is flowing faster.

EVALUATION OF LIQUID VELOCITIES

(i) Consider U (velocity of liquid A)

$$R_x = \frac{-\mu_A\,dU}{g_c dx} \quad \text{(since U decreases as x increases)}$$

Substitute in equation D(1)

$$\Delta P(x - y)b + \frac{\mu_A dU\ Lb}{g_c dx} + R_i \Delta Lb = 0$$

Separate variables and integrate

$$U = \frac{-\Delta P g_c}{\mu_A \Delta L}\left(\frac{x^2}{2} - yx\right) - \frac{R_i g_c}{\mu_A}(x) + C$$

Evaluate the constant C

when x = 2a, U = 0

$$C = \frac{\Delta P g_c}{\mu_A \Delta L}\,(2a^2 - 2ay) - \frac{R_i g_c}{\mu_A}\,(2a)$$

Therefore: $U = \frac{\Delta P g_c}{\mu_A \Delta L}\left(2a^2 - 2ay - \frac{x^2}{2} + yx\right) + \frac{R_i g_c}{\mu_A}(2a - x)$ ..D3

(ii) Consider V (velocity of liquid B)

$R_x = \frac{\mu_B dV}{g_c dz}$ (since V increases as z increases)

Substitute in equation D(2)

$$\Delta P(y - z)b - \frac{\mu_B dV \Delta Lb}{g_c dz} - R_i \Delta Lb = 0$$

Separate variables and integrate

$$V = \frac{\Delta P g_c}{\mu_B \Delta L}\left(yz - \frac{z^2}{2}\right) - \frac{R_i g_c}{\mu_B}(z) + C$$

Evaluate the constant C

when $x = 0$, $V = 0$

$C = 0$

Therefore: $V = \frac{\Delta P g_c}{\mu_B \Delta L}\left(yz - \frac{z^2}{2}\right) - \frac{R_i g_c}{\mu_B}(z)$ ....D(4)

EVALUATION OF THE INTERFACE SHEAR FORCE

when $x = y$ and $z = y$, $V = U$

$$\frac{\Delta P g_c}{\mu_A \Delta L}\left(2a^2 - 2ay + \frac{y^2}{2}\right) + \frac{R_i g_c}{\mu_A}(2a - y) =$$

$$\frac{\Delta P g_c (y)}{\Delta L \mu_B\, 2} - \frac{R_i g_c (y)}{\mu_B}$$

Therefore:

$$R_i = \frac{\frac{\Delta P}{\Delta L}\left(\frac{2a^2}{\mu_A} - \frac{2ay}{\mu_A} + \frac{y^2}{2\mu_A} - \frac{y^2}{2\mu_B}\right)}{\left(\frac{-2a}{\mu_A} + \frac{y}{\mu_A} - \frac{y}{\mu_B}\right)}$$ ....D(5)

EVALUATION OF FLOW RATE

$$Q_A = \int_{x = y}^{x = 2a} bU\,dx \qquad Q_B = \int_{z = 0}^{z = y} bV\,dx$$

CONSIDERATION OF SOME SPECIAL CASES

(i) Ratio of A to B in the pipe is unity

Evaluation of R_i

since $y = 0$

$$R_i = \frac{\Delta Pa}{2\Delta L}\frac{(\mu_A - \mu_B)}{(\mu_A + \mu_B)}$$

Evaluation of Q_A

$$Q_A = \int_{x = a}^{x = 2a} bU\,dx$$

$$= \frac{\Delta Pba^3 g_c}{\Delta L \mu_A 12}\frac{(\mu_B + 7\mu_A)}{(\mu_B + \mu_A)} \quad \text{.....................D(6)}$$

Evaluation of Q_B

$$Q_B = \int_{z = 0}^{z = a} bV\,dz$$

$$= \frac{\Delta Pba^3 g_c}{\Delta L \mu_A 12}\frac{(\mu_A + 7\mu_B)}{(\mu_A + \mu_B)} \quad \text{....................D(7)}$$

If we let the total flow rate be equal to Q then this should equal $Q_A + Q_B$ if μ_A is set equal to μ_B

$$Q = \frac{2\Delta Pba^3 gc}{3\mu\Delta L} \quad \text{..............................D(8)}$$

This equation checks with the development given by Streeter (20).

(ii) Complete distance 2a occupied by fluid A

Evaluation of R_i

$$R_i = - \frac{\Delta Pa}{\Delta L}$$

Evaluation of Q_A

$$Q_A = \int_{x=0}^{x=2a} bU \, dx$$

$$= \frac{2 \Delta Pba^3 g_c}{3 \mu_A \Delta L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots D(9)$$

(iii) Complete distance 2a occupied by fluid B

Evaluation of R_i

$$R_i = \frac{\Delta Pa}{\Delta L}$$

Evaluation of Q_B

$$Q_B = \int_{z=0}^{z=2a} bV \, dz$$

$$= \frac{2 \Delta Pba^3 g_c}{3 \mu_A \Delta L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots D(10)$$

CALCULATION OF Q_A AND Q_B FOR DIFFERENT IN SITU RATIOS OF OIL TO WATER

Evaluation of R_i

$$R_i = \frac{\Delta Pa}{\Delta Lk} \frac{(C_1 \mu_A - C_2 \mu_B)}{(C_3 \mu_A + C_4 \mu_B)} \quad \ldots\ldots\ldots\ldots\ldots\ldots D(11)$$

Where C and k are constants dependent on the in situ oil-water volume ratio. Values of C and k are tabulated below

for various ratios from 0.1 to 10.

TABLE D 1

In situ ratio	k	C_1	C_2	C_3	C_4
10	11	1	100	1	10
5	6	1	25	1	5
2	3	1	4	1	2
1	2	1	1	1	1
0.5	3	4	1	2	1
0.2	6	25	1	5	1
0.1	11	100	1	10	1

Evaluation of Q_A and Q_B

$$Q_A = \frac{\Delta P b a^3 g_c}{\mu_A \Delta L} \frac{(M_1/\mu_A + M_2/\mu_B)}{(M_3/\mu_A + M_4/\mu_B)} \qquad \text{D(12)}$$

$$Q_B = \frac{\Delta P b a^3 g_c}{\mu_B \Delta L} \frac{(N_1/\mu_A + N_2/\mu_B)}{(N_3/\mu_A + N_4/\mu_B)} \qquad \text{D(13)}$$

Where M and N are constants dependent on the in situ oil-water volume ratio. Values of M and N are tabulated below for in situ ratios from 0.1 to 10.

TABLE D 11

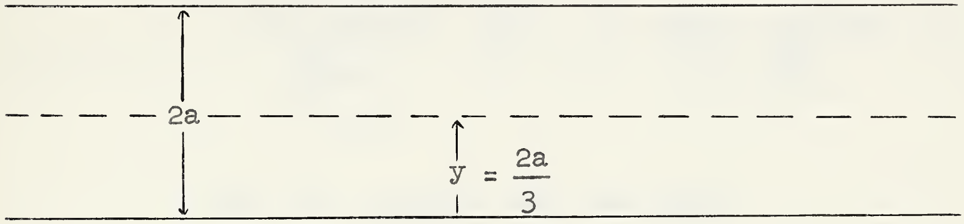
In situ ratio	Q_A				Q_B			
	M_1	M_2	M_3	M_4	N_1	N_2	N_3	N_4
10	2.16	5.1	1	10	0.00035	0.180	1	10
5	1.79	2.02	1	5	0.0031	0.293	1	5
2	1.08	0.42	1	2	0.0247	0.496	1	2

In situ ratio	Q_A M_1	M_2	M_3	M_4	Q_B N_1	N_2	N_3	N_4
1	0.583	0.0834	1	1	0.0834	0.58	1	1
0.5	0.496	0.0247	2	1	0.42	1.08	2	1
0.2	0.293	0.0031	5	1	2.02	1.79	5	1
0.1	0.180	0.00035	10	1	5.1	2.16	10	1

PART 11

SAMPLE CALCULATIONS SHOWING EVALUATION OF Q_A AND Q_B

The following is a series of calculations for an in situ liquid A-liquid B ratio of 2.0.



EVALUATION OF R_i

from equation D(5)

$$R_i = \frac{\frac{\Delta P}{\Delta L}\left(\frac{2a^2}{\mu_A} - \frac{2ay}{\mu_A} + \frac{y^2}{2\mu_A} - \frac{y^2}{2\mu_B}\right)}{\left(-\frac{2a}{\mu_A} + \frac{y}{\mu_A} - \frac{y}{\mu_B}\right)}$$

substitute for $y = \frac{2a}{3}$

$$R_i = \frac{\frac{\Delta P}{\Delta L}\left(\frac{2a^2}{\mu_A} - \frac{4a^2}{3\mu_A} + \frac{4a^2}{18\mu_A} - \frac{4a^2}{9\mu_B}\right)}{\left(-\frac{2a}{\mu_A} + \frac{2a}{3\mu_A} - \frac{2a}{3\mu_B}\right)}$$

simplify

$$R_i = \frac{\Delta P\,(\mu_A - 4\mu_B)}{3\Delta L(\mu_A + 2\mu_B)}$$

EVALUATION OF Q_A

$$Q_A = \int_{x=\frac{2a}{3}}^{x=2a} bU \, dx$$

$$= b\int_{x=\frac{2a}{3}}^{x=2a}\left[\frac{\Delta P g_c}{\mu_A \Delta L}\left(2a^2 - \frac{4a^2}{3} - \frac{x^2}{2} + \frac{2ax}{3}\right) + \frac{R_i g_c}{\mu_A}(2a - x)\right] dx$$

divide the integral into two parts

$$Q_A' = \frac{b\Delta P g_c}{\mu_A \Delta L}\left(2a^2 x - \frac{4}{3}a^2 x - \frac{x^3}{6} + \frac{x^2}{3}\right)_{\frac{2a}{3}}^{2a}$$

$$= \frac{\Delta P b a^3 g_c}{\mu_A \Delta L}(0.79)$$

$$Q_A'' = \frac{R_i g_c}{\mu_A}\left(2ax - \frac{x^2}{2}\right)_{\frac{2a}{3}}^{2a}$$

$$= \frac{b\Delta P g_c a^3}{\mu_A \Delta L}(0.296)\frac{(\mu_A - 4\mu_B)}{(\mu_A + 2\mu_B)}$$

Therefore $Q_A = \dfrac{\Delta P b a^3 g_c}{\Delta L \mu_A}\dfrac{(1.08\mu_A + 0.42\mu_B)}{(\mu_A + 2\mu_B)}$

EVALUATION OF Q_B

$$Q_B = \int_{z=0}^{z=\frac{2a}{3}}\left[\frac{\Delta P g_c}{\Delta L \mu_B}\left(yz - \frac{z^2}{2}\right) - \frac{R_i g_c}{\mu_B}(z)\right] dz$$

$$= b\,\frac{\Delta P g_c}{\Delta L \mu_B}\left(\frac{yz^2}{2} - \frac{z^3}{6}\right)_0^{\frac{2a}{3}} - \left(\frac{R_i g_c}{\mu_B}\frac{z^2}{2}\right)_0^{\frac{2a}{3}}$$

$$Q_B = \frac{\Delta P b a^3 g_c}{\Delta L \mu_A}\left(0.100 - 0.0742\,\frac{(\mu_A - 4\mu_B)}{(\mu_A + 2\mu_B)}\right)$$

$$= \frac{\Delta P b a^3 g_c}{\Delta L \mu_A}\,\frac{(0.0247\mu_A + 0.496\mu_B)}{(\mu_A + 2\mu_B)}$$

EVALUATION OF THE INPUT OIL-WATER VOLUME RATIO, R_V

$$R_V = \frac{Q_A}{Q_B}$$

$$= \frac{\mu_B}{\mu_A}\,\frac{(1.08\mu_A + 0.42\mu_B)}{(0.0247\mu_A + 0.496\mu_B)}$$

PART 111

SAMPLE CALCULATION FOR INPUT OIL-WATER VOLUME RATIO, R_V

μ_A' = viscosity of oil = 18.0 cp @ 25° C

μ_B' = viscosity of water = 0.894 cp @ 25° C

For an in situ oil-water volume ratio of 2.0,

$$R_V = \frac{0.894}{18.0} \frac{(1.08\ (18)\ - 0.42\ (0.894)}{(0.0247\ (18) - 0.496(0.894)}$$

$$= 1.11$$

A series of this type of calculation yields the following set of results.

TABLE D 111

THEORETICAL HOLD-UP

IN SITU OIL-WATER VOLUME RATIO	INPUT OIL-WATER VOLUME RATIO	HOLD-UP
10	13.4	1.34
5.0	5.28	1.06
2.0	1.11	0.55
1.0	0.26	0.26
0.5	0.0532	0.106
0.2	0.00692	0.035
0.1	0.00170	0.0170

PART 1V

CALCULATION OF A PARALLEL PLATE FRICTION FACTOR

From the theory, Equation 10: $f_{pp} = \frac{4}{\rho K\ Q}$. Since for water ρ is known Q must be chosen and K calculated from the in situ oil-water volume ratios.

Table D 1V shows the value of the parallel plate friction factor for the flow rates 1.0 $(10)^{-4}$ ft^3 per sec, 5.0 $(10)^{-4}$ ft^3 per sec and 10.0 $(10)^{-4}$ ft^3 per sec.

TABLE D 1V

PARALLEL PLATE FRICTION FACTOR

IN SITU OIL-WATER VOLUME RATIO	INPUT OIL-WATER VOLUME RATIO	K*	FRICTION FACTOR f_{pp}**		
			Q_1 $1(10)^{-4}$	Q_2 $5(10)^{-4}$	Q_3 $10(10)^{-4}$
10	13.4	9.84	65.2	13.1	6.52
5	5.28	23.6	27.2	5.44	2.72
2	1.11	74.8	8.58	1.72	0.858
1	0.26	178.0	3.61	0.721	0.361
0.5	0.05	377.0	1.70	0.341	0.170
0.2	0.007	696.0	0.921	0.184	0.0921
0.1	0.0017	863.0	0.744	0.149	0.0744
0	0	1095.0	0.586	0.117	0.0586

* $K = \frac{1}{\mu_B} \frac{(N_1\mu_A - N_2\mu_B)}{(N_3\mu_A - N_4\mu_B)}$

** $f_{pp} = \frac{4}{\rho KQ}$ (based on water properties)

NOMENCLATURE

D	=	diameter of pipe, ft
V	=	velocity, ft per sec
$\Delta P'$	=	pressure drop, inches of water
ΔL	=	length of test section, ft
ρ	=	1/v, density, ft^3 per lb
μ'	=	viscosity, cp
R_V	=	input oil-water volume ratio, ft^3 per ft^3
H_R	=	hold-up ratio: ratio of input oil-water volume ratio to the in situ oil-water volume ratio, dimensionless
f_w	=	superficial friction factor based upon water properties, dimensionless
V_w	=	superficial water velocity, ft per sec
f_{pp}	=	parallel plate friction factor based on water properties, dimensionless

NOTE: Additional nomenclature in Appendix D

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